Financial Risk Modelling for Insurers

In a racing car, the driver's strategic decisions, choice of fuel mixture and type of tires are inter-dependent and determine its performance. So do external factors, especially the weather. Like a racing car, an insurance company has a huge number of moving parts, whose functioning is inter-related. The problem for insurers is that these parameters are subject to large amounts of uncertainty, and they have to make choices driving forward at 200mph while checking the rear view mirror. They won't know for more than a year whether they have even completed the race and it might take three years before they know whether they have won.

Today, racing teams model these variables and their relationships for optimum performance. Property-casualty insurers look to financial modeling for the same purpose. The aim of financial risk modeling is to understand the relationships among the risks to the business that arise from its assets, liabilities and current underwriting, all of which are subject to internal decisions and external factors, such as inflation and the legal environment. Such models combine many versions of potential variations in the movement of these variables to give a risk profile of the business. This helps the insurer determine:

1. Capital need and allocation
2. Reinsurance strategies
3. Growth planning
4. Asset allocation
5. Company valuations for mergers and acquisitions

Good vs. weak models

Financial models are data-hungry, complex programs underpinned by very sophisticated mathematical arguments. They produce no magic answers, but there are differences between good models and poorer ones.

A good model will show as realistically as possible the balance between risk and reward from a range of different strategies, which could mean changes to the asset mix, reinsurance program or choice of lines of business to grow, for example. A weaker model will exaggerate certain aspects and under-estimate others. It gives a misleading impression and may result in an overly-adventurous or overly-cautious approach with serious consequences for the insurer's financial performance.

A good model also deals successfully with the inevitable uncertainty affecting the model parameters. Inflation over the next three years could be 3%; it could be 5%, for example. The difference is likely to be material to both sides of the insurer's balance sheet.

In every model, a variety of different risk elements are represented – each with its own set of issues and pitfalls. Many factors - the extent and quality of the data, the assumptions used and the mathematical methods - affect the suitability and usefulness of the model.

The risk is that the user may not realize for more than a year whether the model is a good one for its business. Ultimately, the insurer needs to have confidence in the model maker. Guy Carpenter Instrat® believes that the most important more ▸

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elements that differentiate model quality are these:

- An understanding of developments in mathematical techniques used to express the relationship between risks
- The importance of these risks to the business decision
- Deep knowledge of the fundamentals of those risks
- A trusted relationship with the insurer

Model structure

A good financial model will focus on those business elements that are material to the performance and development of the enterprise and for which there are suitable tools available and which are amenable to a consistent approach. The model can also incorporate the output of other models specific to certain risks or asset classes, such as catastrophe and credit risks, but it needs to take into account the uncertainty associated with those models.

Other, more detailed models and risk management can deal with business issues that pure financial models do not cover in detail. For example, operational risks such as reputational risk, IT hardware and software exposures, pension funding inadequacy, loss of key executives, rogue trader and fraud can be extremely important to the success of a business. In fact, they are more often the cause of defaults by seemingly sound insurers than the specific financial risks of the business.

There are specialized processes to manage each of these operational risks, which are also difficult to incorporate into an overall financial model. They can be modeled as a group using informed judgment to quantify operational risk, but this has a high degree of associated uncertainty. Often, the most reasonable approach is to manage the financial risks through a modeling process, while recognizing that there are some operational risks that are, at best, weakly represented in such a model and that they need other management methods.

Essential elements of the financial risk model are thus:

- Setting capital requirements
- Underwriting risk
- Reserving risk
- Asset risk
- Correlation

Capital allocation / growth strategy

One of the main applications of financial risk modeling is evaluating the relative profitability of business sectors to develop a growth strategy. A method of approach is to allocate capital to each sector and then compare returns on allocated capital. Methods of capital allocation and performance measurement vary, and the model needs to be able to allow for their strengths and weaknesses. The remainder of this paper will discuss the details of the model elements needed for a good financial risk model and how such a model can most appropriately be used to develop a growth strategy.

The Elements

Setting capital requirements

Insurers need capital to sustain their current underwriting, provide for the possibility of adverse reserve changes, and support business development. Regulators and rating agencies want insurers to retain generous amounts of capital, but shareholders want capital used efficiently. Financial modeling should help the insurer balance these opposing interests to establish the optimum level of capital that achieves both efficiency and prudence.

The approach taken by some models to setting capital requirements is default avoidance, based on either the probability of default – total loss of capital - or the economic value of the possibility of default. Default is not the most germane reference, however, because most insurers want to stay in business even after quite adverse events, and so they need to maintain a significant level of capital.

The franchise value of an insurer includes the imbedded value of the renewal book of business, which is typically significantly more profitable than new business. To continue to write this business might take 80% of current capital levels, in which case the relevant reference point for capital need is a loss of 20% of capital, not default.

In addition, exhausting all the capital is an extreme outcome that is difficult to model accurately. To generate default would normally take events far in the tail of the financial risk distributions where they are poorly understood. The assumptions made about the form of the distributions have a big impact here, but there is little data in the tail. Also, for a basically sound insurer to default within a one or two year time-frame is likely to be the result of operational risk that is not well modeled either. Short-term default probabilities are basically guesses projected way beyond any data by means of assumptions.

Thus, defining capital requirements so that no more than 20% of capital, or something similar, would be lost in an adverse year is both more feasible to measure and more relevant to business needs than the probability of default. For example, capital requirements could be set so that the average loss in the 50-year or worse scenarios leaves 80% of surplus. Taking a "target-level-or-worse" average reflects the fact that when the target level is reached, the amount that it is exceeded by is a random event. A useful starting point is to find the return period on this basis for retaining enough of the current capital to support the renewal book of business.

Underwriting risk

Quantifying loss potential is an essential element to modeling underwriting risk and approaches. Some of the issues that must be addressed to properly model underwriting risks include:

Loss frequency and severity distributions

- Mispricing
- Parameter risk
- Catastrophe modeling uncertainty more ▸

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A critical area of focus is parameter risk, which is often the biggest risk next to catastrophe exposure and may even be bigger than catastrophes on a net basis.

Loss frequency and severity distributions

A variety of frequency and severity distributions that fit the unique characteristics of insurance loss data are now available, based on Venter in *Transformed Beta and Gamma Distributions and Aggregate Losses* (1983) and reformulated by Kreps in the Instrat® working paper Continuous Distributions (1998). Modern statistics also provides good methods of estimating parameters, testing the quality-of-fit and understanding the uncertainties that remain. Good control of these issues, however, is still a differentiating feature of the best modellers.

Mispricing

Losses are not the only underwriting risk. No matter how severe losses are, inadequate premiums will exacerbate their impact on profitability, and under pricing may be difficult to ascertain for a few years, creating a significant cumulative effect.

Parameter risk

This describes all risks of underwriting exposure not coming from known fluctuations in frequency and severity. Aspects of parameter risks include:

- Estimation risk
- Projection and event risks
- Systematic risk

Estimation risk

Quantifying frequency and severity requires data, and there is never enough to know true probabilities. More and better data reduce the risk but it never disappears.

Statistical methods quantify how far the estimated parameters can be from true, and this can be used in scenario generation by modeling the probability that any given set of parameters is the correct one.

Projection and event risks

Projection risks refer to changes in risk conditions that arise from uncertainty about developing trends, for example:

- Driving increases because gas is cheaper
- Criminals attack security vehicles because banks are more secure
- A long-term shift to more extreme weather events aggravate property damage

Event related risks encompass situations where there is a clear, casual link with a large unpredicted event and a change in circumstances. They usually result from unexpected changes in circumstances outside the company's control and can affect the frequency or severity of losses. For example:

- A court finding favors a large group of policyholders.
- A new cause of loss emerges that was previously regarded as not covered.
- The regulator bars an important exclusion.
- A new entrant into the market reduces rates to grab market share.
- A rogue trader violates policy and heavily bets company assets.

These are large risks in monetary terms and they can dwarf others, as the continuing asbestos saga demonstrates. They are hard to predict but need to be treated as a continuing source of risk and included as variables in the range of potential scenarios.

Systematic risk

All parameter risks can be regarded as systematic in the sense that they do not improve by diversification – i.e. by adding volume. Some of these are macro-economic factors, of which inflation is the most important. It can affect most liability and many asset classes.

For example, consider an automobile insurer with 200,000 expected claims. Even with a heavy-tailed severity distribution (standard deviation seven times the mean), the loss ratio would be quite stable in the absence of parameter uncertainty. For an expected loss ratio of 65%, the 1-in-10 year loss ratio would be 66.3% and the 1-in-100 would be 67.4%. This is more stable than is realistic in practice. Adding a 5% parameter risk would increase the 1-in-10 ratio to 69.4% and the 1-in-100 to 73.3%, which are more reasonable, although perhaps still a bit too stable.

Model uncertainty

Catastrophe models are a further source of uncertainty. The providers' models differ not only from each other, but also from themselves over time, as the modeling companies frequently release updates. This variability reflects the fact that the models contain considerable uncertainty, first relating to hazard probability and possibly even more in the amount of insured damage likely to result from a specific event. Further uncertainty in the output results from data quality, including a mismatch of company data fields and cat model assumptions. An asset liability model, therefore, needs to include the risks related to catastrophe model results, which can in part be quantified by the use of multiple models.

Loss reserve risk

The risk of reserves running off other than as anticipated is significant for property-casualty insurers and is easy to understate, both in terms of the time capital has to be held and the amount that should be held for loss reserves and unearned premiums reserves (UEPR).

Among the points that need to be established to measure reserve risk are:

- Do the losses that emerge in a period depend on the losses already emerged?
- Is all loss emergence proportional to earlier losses?
- Is emergence independent of calendar year events?
- Are the parameters stable?
- Are the disturbance terms generated from a normal distribution?
- Do all the disturbance terms have the same variance? more ▸

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Actuaries usually start with development factors and the Bornheutter method, which is used to model loss reserves for slowly developing situations where the first one or two years of results do not give a good indication of the likely results. This approach has its limitations and other techniques are also needed. A discussion of methods of modeling reserve risk, including testing the points listed above against the data being used, is covered in Venter *Testing the Assumptions of Age-to-Age Factors* (1998). Specifying a model for reserve runoff and testing it with quality-of-fit measures are key aspects of this approach.

Asset risk

Insurers in different segments of insurance in different regions take different approaches to investing. Modeling of the asset risk will need to emphasize the right issues in each market. The main asset classes are equities and bonds, but real-estate linked investments are important in some markets. Different types of bonds are important in different regions. Modeling of foreign exchange risk and inflation risk are closely related.

A key aspect of asset modeling is probabilistic reality: modeling scenarios consistent with historical patterns. Asset modelers try to generate a large variety of scenarios against which to test the insurer's strategy, but variety alone is not enough for proper risk assessment. The more probable scenarios should be given more weight, while the less likely ones should be present according to their probability of happening. This can be evaluated to some degree by looking at historical patterns.

Modellers also can address the balancing of asset and underwriting risk. Because insurers have different risk profiles on the liability and asset sides of their balance sheets, it is possible for them to optimize the use of capital by offsetting insurance risks with investment risk. This works especially well for life insurance companies, in that the matching investments often have high returns. Property-liability companies can lose out on potential return by perfect matching, while modeling may show that they can adopt a more risky overall strategy that has better return characteristics with overall risk still at manageable levels.

A traditional tool for asset risk analysis is the mean-variance efficient investment frontier, which should be supplemented with ranges of return around the means. Financial risk modeling also allows analysis of how the efficient frontier of total return changes with different reinsurance programs. The reinsurance program can be adjusted to fit best with the investment portfolio or the asset mix itself can be altered, according to conditions in the reinsurance and financial markets.

Bonds

One way to eliminate unlikely scenarios in the bond market is to use arbitrage-free models. Although some arbitrage is often possible with published yield curves, these possibilities tend to be fleeting and hard to exploit. Having such scenarios in a bond model could misdirect choices towards strategies that ostensibly make arbitrage profits which would fail in practice. The model should also capture historical features of the bond markets, like high autocorrelations and distributions of yield spreads. This is discussed in detail in Venter *Testing Distributions of Stochastically Generated Yield Curves* (2003a).

Equities

There is a degree of correlation between bond and equity returns that should be incorporated into the scenario generation. A starting point for equity modeling is the geometric Brownian motion model underlying the famous Black-Scholes options pricing model. However, the implied volatilities of options in the market do not relate to each other as this model predicts. Models that allow for more extreme motion of equity prices, or even discontinuities, would be more realistic.

Foreign exchange

There are historical relationships between a country's interest and exchange rates, and these can be used to formulate a foreign exchange model. Economists have found that changes in actual and anticipated interest rates in two countries lead to changes in the exchange rates. However, to model this accurately, it is not enough to separately model the interest rates in the two economies and then forecast exchange rates based on that. The interest rate movements across different economies are themselves correlated, and the model should build this in before creating the resulting exchange rate scenarios.

Correlation

Exploring correlations and their potential effects are fundamental to the financial model, but there are different mathematical approaches to correlation and the method chosen by the modeller will determine how closely the output approaches the probable scenarios, especially in extreme situations.

Correlation issues infuse financial risk modeling. Losses across lines can be correlated, as well as losses with assets and assets with each other. The correlation between inflation and development is another key issue.

As the severity of events increases, so does the probability of losses across multiple lines of business - what is known as tail dependency. Thus, the model needs to capture the likely level of correlation across the company's book in case of a catastrophic windstorm, earthquake or an event, such as the World Trade Center. In a large earthquake, for example, large losses to property, workers' compensation and automobile lines are highly probable, while they are not otherwise particularly correlated except by inflation.

To quantify the potential impact in an extreme event, the model needs to identify the degree of correlation, the part of the spectrum which is correlated and the level of probability. How well it can do so depends on the ability of the statistical method chosen to extrapolate to extremes. It is important in modeling dependencies to try to model from the physical cause and not just try to stuff data into a convenient mathematic form! What is typically desired here is a method that will generate correlations in large losses but not small.

The usual practice of applying correlation through the multi-variate normal distribution does not capture this effect, more ▸

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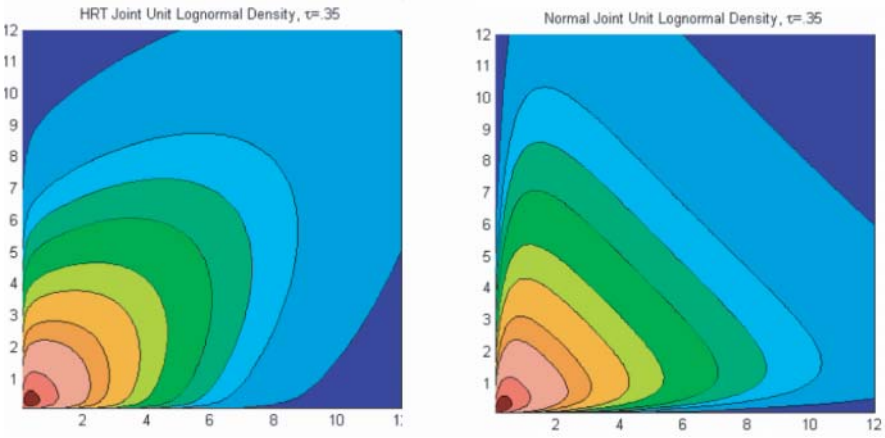
but there are useful alternatives, for example, copulas, which express the relationship between two or more random variables. There are different types of copulas and the choice of copula is critical in determining how well the model describes the likely impact of an extreme event on a portfolio. Some are much more effective than others at bringing out the potential correlations at the extreme. Venter *Tails of Copulas* (2002) and *Fit to a t – Estimation, Application and Limitations of the t-copula* (2003b) develop methodology for using copulas.

Exhibit 1 graphs two copulas that have the same degree of overall correlation. The one on the left has a stronger relationship in the upper right corner, which is the region where losses are large for both lines.

Exhibit 1 Variations in the Correlation of Extreme Events



Capital allocation / growth strategy

One of the applications of financial modeling is to identify the higher-return lines in order to develop and refine the growth strategy. Some companies like to allocate capital to each line or unit, and then compare returns on the allocated capital. This is not the only way to approach this issue, however. Several alternative approaches are outlined in Venter *A Survey of Capital Allocation Methods with Commentary* (2003c). These are discussed briefly below.

Sometimes in this process, analysts allocate some capital to the investment function as well. It is useful to have measures of the riskiness of investments vs. underwriting, so developing consistent risk measures to apply to them is instructive. But for measuring relative performance and setting growth strategy, it should be recognized that each business unit generates investment returns on cash flow and capital supporting the business. That income and the capital to support those investments need to be charged to the unit for a proper evaluation of its economic contribution to the business as a whole.

Alternatives for capital allocation and performance measurement

Four methods of performance measure are discussed:

- Allocate capital by means of a risk measure.
- Allocate capital by the market price of bearing risk.
- Charge capital costs directly against profits.
- Compare the value of float to a leveraged mutual fund.

Allocate by risk measure

This requires selecting a risk measure and then an allocation method. A number of popular risk measures are based on tail risk:

- Value at risk (VAR) looks at the loss at a reasonably high probability level, say 1-in-100.
- Tail value at risk (TVAR) looks at the average loss excess of the selected probability level, which recognizes that the adverse loss will not necessarily be right at the selected level.
- Weighted tail value at risk (WTVAR) is similar but it weights the more adverse scenarios more heavily, as risk aversion tends to increase more than linearly with loss size.

If WTVAR is weighted by performing a transformation on the loss probabilities, it can be tied in to risk pricing methods to make this risk measure represent the market value of the excess losses. The minimum martingale measure of Møller *Stochastic orders in dynamic reinsurance markets* (2003) provides an example of such a transformer that seems to work well in practice.

Assuming that premiums are enough to cover expected losses, then capital would be needed for the portion over the mean. This can be reflected in the tail measures by subtracting the mean loss from the measure. An X added to the name of the measure will denote that it is excess of the mean. Thus WXTVAR would be the weighted tail losses less the mean.

There are also a number of possible allocation methods, such as spreading in proportion to marginal contribution to company risk. Kreps Instrat® working paper *Riskiness Leverage Models* (2003) presents a method of creating additive co-measures for almost any risk measure. These allow a completely additive allocation of capital in a manner consistent with the original risk measure. Co-WXTVAR meets a lot of objectives of an allocation method and works well in practice too. The excess more ›

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point has to be determined, but the loss level that produces an operating loss would be a reasonable starting point.

Allocation by risk measure has drawbacks. The choice of measure and method is arbitrary, business units will argue for choices that favor them and there will be no underlying theory to provide integrity. Also, if pricing is calculated so as to equalize returns on allocated capital, it may not conform to risk pricing standards. For example, correlation with the market might be an element of the market value of carrying risk, but this usually is not considered in allocating capital by risk measures.

Allocate by price of bearing risk

Financial theory gives market price guidelines for risk bearing and these can be calibrated to the insurance market situation. Business units can then be evaluated by profit vs. risk-pricing standards. The business can allocate capital to units in proportion to target profitability, which would then make a constant return on capital across units an appropriate goal. This gets around the problem that allocation by risk measure might require different returns on allocated capital by line. There is a lot of work needed to get this right, however. Capital market pricing methods are still undergoing development, and their application to insurance has unique issues, like the value of non-market risk, heavy-tailed distributions and sudden volatility in results.

Charge capital costs against profits

This method involves subtracting the cost of capital from the unit's profitability, instead of dividing the profit by the capital itself. It should use the true marginal capital costs of the business being evaluated instead of a proportion of the entire firm's capital. For example, to evaluate the impact of growing the business by 10%, the charge would be the cost of the capital needed for that much growth. Likewise, if the question is one of ceasing to write a line, then the cost should be that of the amount of capital saved. Thus, profits are compared to marginal costs. This is a fundamental principle of market pricing.

Value of risk guarantee of parent

One way to evaluate the marginal cost of capital of a business unit is to set it at the value of the guarantee the overall business makes to the policyholders of the unit. Essentially the parent is guaranteeing to pay up if the unit runs out of funds. This guarantee is a complex option in which the unit first pays out its premiums and any investment income on it, and if that does not cover the losses, the parent pays all remaining losses. Options pricing methods can be used to compute the value of this option. It implicitly costs the parent that much to provide the guarantee, so the value of the profits should be at least that much.

Comparison of value of float generated by the business to a leveraged investment fund with the same risk

An insurer can be regarded as a tax-disadvantaged leveraged mutual fund. A mutual fund does not have to pay tax on its earnings – only its customers do. But it only has the provided capital to invest. An insurer has more assets than capital, so is leveraged from an investment fund viewpoint. It should be possible to specify a leveraged mutual fund with the same distribution of returns as the insurer by establishing a borrowing rate, an amount borrowed and an investment portfolio. If the result is a fund that would have to borrow high amounts at unrealistically low rates, then the insurance business is adding value. Business units can be evaluated by their marginal impact on the borrowing rate.

Comparing the methods:

- Allocation by risk measure is straightforward but involves arbitrary choices and is unlikely to produce a constant proper return rate across businesses.
- Risk pricing is appropriate for comparing profitability, but has some unsolved problems associated with it.
- Comparing profit to the actual marginal surplus cost directly determines the economic contributions of business units. This is not the same as allocation in proportion to marginal risk.
- Leveraged mutual fund comparison is appropriate for evaluating the total value-added of the business and the marginal contribution of each business unit to it. ■

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