Strategic Planning Models

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ERM has gained a good deal of momentum from regulatory interest. However the ability to model a wide range of risks a firm faces, and to do so with consistent metrics across risks, also provides a platform for analysis of strategic options. This may lead to a somewhat different choice of risk metrics and modeling approaches than those in a regulatory-focused model.

One fundamental step in strategic modeling is to develop methods for quantifying risk-adjusted profitability by business segment. This can give an indication of which segments that are worth developing further as well as the segments that may require re-structuring. Two basic approaches to risk-adjusting profits are explored here: capital allocation and capital consumption. But to address these properly, the overall capital requirements of a company and choices of risk measure need to be touched upon as well.

Overall Capital Requirements – Beyond Economic Capital

One of the typical outputs of financial risk models is economic capital. That is usually specified as a remote percentile of the probability distribution of outcomes – perhaps the 1-in-2500 or 1-in-3333 point on the distribution. Economic capital can be useful for regulators, by showing how bad results could get. It actually reveals very little information about the full probability distribution, so under the assumption that regulatory information becomes public information, it does not inform competitors very much. This combination makes economic capital useful as a regulatory standard.

Economic capital can be estimated by a simulation approach, or even by estimating the mean and variance of outcomes and assuming a distributional form. Usually the particular percentile used is selected to be a round number that is as high as possible under the constraint that economic capital is less than actual capital. Then the company will appear to be financially sound by the selected criteria, which in turn will look appropriately conservative.

The advantages of economic capital for regulatory purposes may be detriments for strategic planning, however. For strategic planning, company management may want to look at more than a single percentile of its distribution of outcomes. Having more capital than is economically needed may not be acceptable to owners, either. They usually want their target return to be met on total capital, not some smaller amount. Also there is little to support the idea that a percentile so selected will be the right capital level for a company.

An alternative approach to capital requirement is setting capital at the level that maximizes the franchise value of the firm, defined here as the excess of the market valuation over the capital held. This target for capital does not necessarily appear by inverting a probability distribution function. Determining it may require a more detailed analysis of business opportunities available, effect of capital levels on attracting and retaining customers, etc. Whatever it is determined to be, the need for an extra margin beyond that would depend on the company's relationship with capital providers. If a company has a close relationship with its investors, it may be able to give them back any temporarily unneeded capital with the confidence that it will be re-supplied if necessary. More commonly, however, a company that finds itself needing capital may be able to raise it only at a good deal of expense. In that case, more than the otherwise optimal amount may be worth retaining in order to absorb possible adverse fluctuations. But then the company will still be held accountable for return on the total.

Relating Capital to Risk Measures

Economic capital is a risk measure of company results, also called value-at-risk (Var). It is set at such a high probability level that it is difficult to measure with any degree of confidence, however, under the current state of risk models. Actual capital could also be expressed as Var at some high percentile, but again there could be a good deal of doubt about what percentile that actually is. The advantage of economic capital is that the reciprocal of the probability is a round number. For strategic planning purposes the advantage of actual capital is that the investors will typically want to see what the return is to actual capital. Holding unneeded capital for investment only is not usually popular outside of Omaha.

An alternative to Var at a remote probability is to express capital as a multiple of a risk measure that is more readily calculated. Var at a lower percentile is one such possibility. So is standard deviation, or semi-standard deviation. These have been criticized as missing skewness risk, since they are based on second moments only. A moment-like risk measure that is responsive to heavy tails is $E(Xe^{cX/EX})$ for some value of c.

A popular tail-based measure is tail value at risk (Tvar), sometimes called conditional tail expectation (CTE). It is the average of all losses beyond the selected percentile. For example, if there is a 15% chance the company will suffer a loss, then Tvar at the 85^{th} percentile would be the average loss when there is a loss. Then capital can be expressed as a multiple of that risk measure. For instance, on the average when there is a loss, 10% of capital might be lost. Then capital can be allocated as 10 times the allocated Tvar at 85%. Sometimes Tvar at much higher percentiles is used, but that runs in to the calculation problems of economic capital, as well as ignoring important but less severe risk at lower probability levels. Tvar has also be criticized for its linear treatment of large losses, which is in violation of the idea that losses get more painful as they get larger.

While there are many possible risk measures, one promising family is risk measures based on transformed probability distributions. If more probability is given to the adverse outcomes and less to the favorable ones, the result is a riskier distribution. The transformed mean is a risk measure which is more adverse than expected results. Measures like Tvar can be done with transformed probabilities as well, which then does give more weight to the larger losses. Classical risk pricing formulas like Black-Scholes and the capital asset pricing model can also be expressed as expected values under transformed probabilities, so this type of risk measure has potential to represent the market value of the risk being measured. That would be a useful comparison to actual profits achieved. In fact the very idea of comparing profit to risk suggests that the risk measure corresponds to the economic value of the risk being taken. Thus the choice of risk measures to use should reflect the relationship of risk to value.

One popular probability transform is the Esscher transform with parameter h. For a continuous random variable X, its density f(x) is transformed to $f^*(x) = f(x)e^{hx}/Ee^{hX}$, assuming this exists. If X is a compound Poisson distribution with frequency λ and severity g(y), its Esscher transform can be shown to be given by $\lambda^* = \lambda Ee^{hX}$, and $g^*(y) = g(y)e^{hy}/Ee^{hY}$. While typically used in pricing, these transforms can also be used as risk measures to quantify the riskiness of a variable X and the value of the risk.

Allocating Risk Measures

Once capital has been expressed as a multiple of a risk measure, it can be allocated to business segment by so allocating the risk measure. One way to do this is to calculate the risk measure for each segment and allocate proportionally. This does not show the contribution of the business segment to the overall company risk measure, however. The method of co-measures is designed to do this, and it is applicable to many, but not all, risk measures. Co-measures generalize the idea of covariance. Just like the covariances of the segments with the total company sum up to the variance of the company, the sum of the co-measures will be the total risk measure.

If Y is the variable for the results of the whole company and is a sum of the results of the business segments X_j, and the risk measure $\rho(Y)$ can be defined as:

$E[h(Y)g(Y) \mid \text{condition on } Y]$, where h is additive, i.e., $h(U+V) = h(U) + h(V)$.

Then the co-measure for X_j is $r(Xj) = E[h(Xj)g(Y) \mid \text{condition on } Y]$. These add up over the segments to $\rho(Y)$.

For instance, $Var_\alpha(Y) = E[Y \mid F(Y) = \alpha]$, so $r(Xj) = E[Xj \mid F(Y) = \alpha]$ is co-Var. This is a completely additive allocation of Var, and the allocation to each business segment is its contribution to the overall Var. In fact this is more of a decomposition of Var to its component pieces than an allocation, which implies a somewhat arbitrary process.

Common risk measures like standard deviation, Tvar, etc. all have such co-measures. For example, $\text{Tvar}_\alpha(Y) = E[Y|F(Y) \geq \alpha]$, so $r(Xj) = E[Xj|F(Y) \geq \alpha]$ is co-Tvar.

Marginal Decomposition

Even though co-measures are always additive, they are not always marginal. The (incremental) marginal impact of a business segment on a company risk measure is the decrease in the overall risk measure that would result from an incremental proportional decrease to the business segment. That marginal impact is the contribution of the increment to the overall risk measure. It can happen that all these marginal impacts sum up to the total company risk measure. In that case the allocation is called a marginal decomposition of the risk measure.

If profit is compared to marginal impact, this is consistent with the economic principle of pricing in proportion to marginal costs. Another advantage of marginal decomposition is that if the risk-adjusted return is the profit of the segment divided by its marginal risk, then growing the units with the highest risk-adjusted profit will increase the risk-adjusted profit for the company as a whole. This a desirable feature of risk-adjusted profit, but is not guaranteed for allocations other than marginal decomposition.

For a risk measure to have a marginal decomposition, it has to be scalable. That is, $\rho(aY) = a\rho(Y)$ for any constant a. Standard deviation, Var, and Tvar all are scalable, but variance is not. For a scalable measure $\rho(Y)$, the marginal decomposition is defined as: $r(X_j) = \lim_{\varepsilon\to 0}[\rho(Y+\varepsilon X_j) - \rho(Y)]/\varepsilon$.

A theorem of Euler's shows that these do add up to ρ. For instance for standard deviation, it is not difficult to show that $r(X_j) = \text{Cov}(X_j,Y)/\text{Std}(Y)$ is the marginal decomposition. This is a co-measure. The co-measures above for Var and Tvar turn out to be marginal as well, as does any risk measure defined as a transformed mean.

Thus if risk-adjusted profit is based on allocating capital, there are advantages to allocation by marginal decomposition of a risk measure, and using a risk measure that reflects the value of the risk. There is no need to restrict attention to a single risk measure, either, as the different perspectives from different risk measures may prove worthwhile.

Capital Consumption

Allocating capital is not the only way to risk-adjust profits, however, and it has some inherent disadvantages. First of all, it is artificial, in that a segment is not limited to the capital allocated to it, and may in fact use up the entire firm capital if its results are unfavorable enough. Also it is arbitrary, in the sense that there is no one canonical risk measure that stands out.

Capital consumption is one way to get around these problems. Instead of computing a return on allocated capital, an imputed cost of capital is subtracted from segment profits to get the value added of the segment. The cost of capital imputed to the segment is the value of its right to call on the assets of the company in case it loses money on its own. The company is implicitly providing the segment with that option, so the value of the option is the cost to the company of supporting that business segment.

This is a somewhat complex option to evaluate, as there are no time or cost constraints on the payout. Whenever the segment needs the money it can take it, and continue doing so as its cash flow demands. Moreover, the profit of the segment can also be considered an option: the company takes all the profit, if there is any. Thus the value added of the segment is the difference in value between these two contingent claims.

Further Reading

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