Note on Murphy WAD

For an accident year, denoting latest cumulative by y and previous cumulative by x, the model is $y = bx + x^{1/2}e$, where b is the development factor. The residuals e for a column are as usual assumed to be independent across accident years.

Dividing by $x^{1/2}$ the factor b is estimated by least squares as Σy/Σx:

let $u = y/x^{1/2}$, $v = x^{1/2}$. Then: $\hat{b} = \frac{\Sigma uv}{\Sigma v^2} = \frac{\sum \frac{y}{\sqrt{x}}\sqrt{x}}{\Sigma x} = \frac{\Sigma y}{\Sigma x}$.

For a no-constant regression, $\hat{s}^2 = \frac{1}{n-1}\sum e_i^2$ and $V\hat{a}r(\hat{b}) = \frac{\hat{s}^2}{\sum v_i^2}$. In this case,

$V\hat{a}r(\hat{b}) = \frac{\sum x_i\left(\hat{b} - {y_i}/{x_i}\right)^2}{(n-1)\sum x_i}$ and $\hat{s}^2 = V\hat{a}r(\hat{b})\sum x_i$.

This can be done for each column. For the final, say I^th^, column, Mack suggests

$\hat{s}_I^{\,2} = \min\left(\hat{s}_{I-1}^{\,2}, \hat{s}_{I-2}^{\,2}, {\hat{s}_{I-1}^{\,4}}/{\hat{s}_{I-2}^{\,2}}\right)$ and then $V\hat{a}r(\hat{b}_I) = \frac{\hat{s}_I^{\,2}}{x_I}$. This will be assumed to work for Murphy as well.

Murphy calculates the variance of the prediction of ultimate loss as the sum of parameter risk plus process risk. This needs an additional assumption: the residuals e from all the column regressions are independent. He denotes the observation for the latest accident year at lag 0 as $x_{0,0}$, and the estimated losses for all accident years through lag n as $\hat{M}_n$. The error in the estimate of this mean is what he calls parameter risk at lag n. He denotes this as $Var(\hat{M}_n)$, which he calculates recursively for each lag. There are typos in his formulas but from his derivation in the appendix the result can be seen to be:

$$Var(\hat{M}_1) = x_{0,0}^2 V\hat{a}r(\hat{b}_1),$$

$$Var(\hat{M}_{n+1}) = (\hat{M}_n + x_{n,n})^2 V\hat{a}r(\hat{b}_{n+1}) + [\hat{b}_{n+1}^2 + V\hat{a}r(\hat{b}_{n+1})]V\hat{a}r(\hat{M}_n).$$

Process risk is the variance of the actual emerged loss around its mean. For lag n he calls the difference from the mean S_n, and its variance is also calculated recursively:

$$V\hat{a}r(S_1) = x_{0,0}\hat{s}_1^2,$$

$$V\hat{a}r(S_{n+1}) = (\hat{M}_n + x_{n,n})\hat{s}_{n+1}^2 + \hat{b}_{n+1}^2 V\hat{a}r(S_n).$$