Capital Consumption, Riskiness Leverage, Co-measures, and the M Curve

The **capital consumption** paradigm is that capital is indivisible so cannot be meaningfully allocated to business units, but the cost of the capital a business unit is using can be calculated. This is set as the cost to the firm of the unit's right to access the firm's capital. Then although it is not possible to compute a return on capital for a business unit, there is still a hurdle rate of sorts in that the profits should exceed the unit's cost of capital[1].

Capital consumption appears to be set in an options-pricing framework, but the options involved are complex, multi-period, firm-specific and non-Gaussian. The pricing of the unit's right to access firm capital has sometimes been formulated using what we call **riskiness leverage functions**. Essentially a big loss is worse than a small loss, usually more than proportionally. This can be expressed with a function g(x) that gives a valuation of the loss x. If the losses and expenses for a particular book of business exceed the premium and investment income on cash flow for the unit, a capital attachment is required. The mean of the leverage function of the capital attachments can be used as a measure of the cost of the capital consumed.

The **M curve** measures the sensitivity of the market-to-book ratio to the capital level, assuming a set of pre-existing obligations and business opportunities. A capital loss of x can produce a reduction in value of more than x, and the extra reduction in value is more than proportional as losses get large. This is because loss of capital reduces business opportunities. The capital loss would be from insurance losses, so the reduction in value from a loss of x can be considered as a greater loss to value of g(x). Thus although the M curve is a function of capital, it leads to a function of losses that is basically a riskiness leverage function.

Co-measures allocate risk measures to business units or even to policies based on the units' contributions to the risk measure. In simulation this contribution is calculated for each scenario and the co-measure is the average of these contributions over all the scenarios[2].

---

[1] There are details here not yet specified, like is it the expected profit that has to exceed the cost of capital, or some other measure of profit?

[2] This assumes that the risk measure for the company is the average of the risk measures for the scenarios.

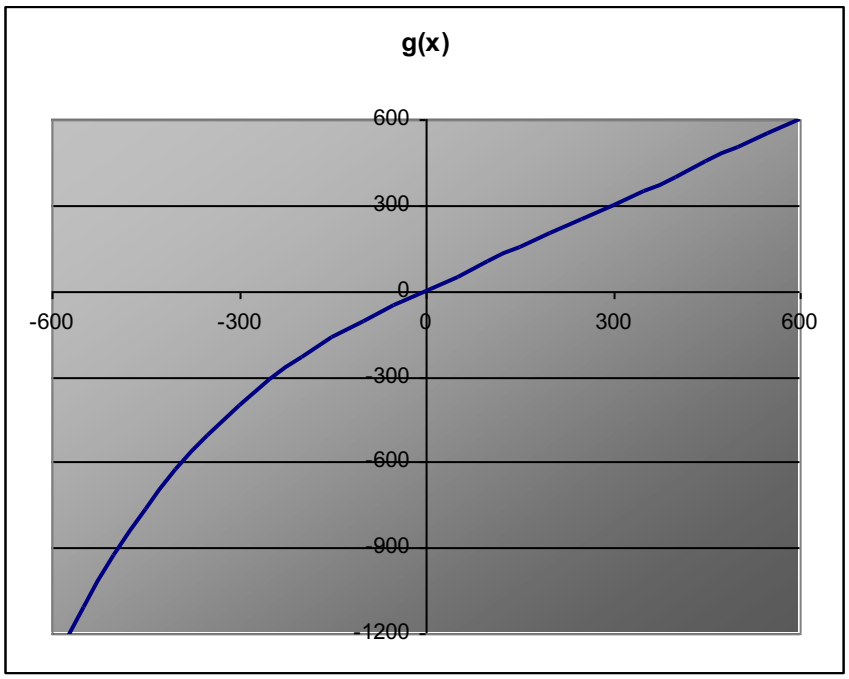
So how can we put all this together? Basically it can be done by using the M curve as the riskiness leverage function and allocating that by the co-measure approach to calculate the capital consumption charge.

The M curve can be formulated as an aggregate measure, used on annual or quarterly aggregate losses. For each aggregate scenario producing a capital loss, the loss in overall firm value can be calculated from the M curve. That value loss can be allocated to business units in proportion to the units' economic losses in the period. The allocation would then be the capital charge to the unit for the scenario. Essentially the charge for a given amount of capital attached would be higher when the capital is also needed for other losses. A business unit that had a profit in a scenario where the company had a loss would get a credit (negative charge) for that scenario, as it would be a capital provider not a capital consumer in that case. A unit that had a loss when the overall company had a profit would have to be charged for that loss but at a lower leverage ratio corresponding to the value of positive earnings, which with some leverage functions could be the earnings themselves, that is, $g(x) = x$. The average of its scenario capital charges would be the overall cost of capital for each unit.

What is the profit contribution that this capital charge should be netted against? If you look on the business unit as having an unlimited option on the firm's capital, so it gets zero capital if it makes a profit and gets all the capital it needs if it makes a loss, then the firm has a similar option on the business unit's profit. It takes all of the profit if there is any, otherwise none. But this profit has to be put through the riskiness leverage function to get the value of the contribution. For scenarios where firm profits in total are negative, any profit the unit earned has already been given a negative capital charge. Thus only the profits in scenarios where the firm and the unit both have positive profits need a profit contribution.

The net contribution of a unit is then its profit contribution less its capital charge. Here is an example.

Let $g(x) = x$ if $x>0$ and $g(x) = x - 2E\text{-}04x^2 + 3E\text{-}06x^3$ if $x<0$ be the leverage function. This g is x for profits and close to x for small losses but gives increasing penalties for larger losses.



For four scenarios and three lines, economic losses are shown below.

Scenario:	1	2	3	4	Average
Gross Economic Profit					
Homeowners	200	-500	150	200	12.5
Comp	100	-100	-50	100	12.5
Auto	100	50	-50	-50	12.5
Total	400	-550	50	250	37.5
Value Change	400	-1110	50	250	-102.4
Capital Charge					
Homeowners	0	1009	0	0	252.2
Comp	0	202	50	0	62.9
Auto	0	-101	50	50	-0.2
Profit Credit					
Homeowners	200	0	150	200	137.5
Comp	100	0	0	100	50.0
Auto	100	0	0	0	25.0
Contribution					
Homeowners	200	-1009	150	200	-114.7
Comp	100	-202	-50	100	-12.9
Auto	100	101	-50	-50	25.2
					-102.4

The value change is the leverage function applied to economic profit for the firm as a whole. This value change is then allocated to line as either a capital charge or profit credit. When either the business unit or the firm has a negative economic profit, the capital charge for a unit is the negative of its economic profit times g(x)/x, the firm's ratio of value change to economic profit.

The profit credit is the same calculation when both the business unit and the firm

have economic profits. The overall contribution of each unit to the firm is then its profit less its capital charge. In this example all the units had expected profits, but the change in expected firm value is negative due to the high risk. Since auto made a profit in scenario 2 when the other lines had big losses, it was contributing significant value to the firm, and got a large capital credit for so doing. This made the line a significant contributor to the bottom line value, not primarily for its profit level but for its counter-cyclical behavior. Nonetheless the firm as a whole is not adding value.

The same calculation can be done net of reinsurance.

		Net			
		Economic Profit			
Scenario:	1	2	3	4	Average
Homeowners	60	-100	10	60	7.5
Comp	75	-15	-85	65	10
Auto	100	50	-50	-50	12.5
Total	235	-65	-125	75	30
Value Change	235	-67	-134	75	27.3
		Capital Charge			
Homeowners	0	103	-11	0	23.0
Comp	0	15	91	0	26.6
Auto	0	-51	54	50	13.1
		Profit Credit			
Homeowners	60	0	0	60	30.0
Comp	75	0	0	65	35.0
Auto	100	0	0	0	25.0
		Contribution			
Homeowners	60	-103	11	60	7.0
Comp	75	-15	-91	65	8.4
Auto	100	51	-54	-50	11.9
					27.3

The reinsurance has cost 20% of the economic profit in terms of margin (40% for homeowners). However it reduces the losses in the bad scenario enough to produce an expected net increase in value. The capital charges and profit credits are calculated as before. The auto line is not contributing as much now because the reinsurance is providing the capital in the bad scenario. It is still the most profitable line, however. Now all the lines are making a positive contribution.

In practice it may be possible to back into the riskiness leverage function for a firm by looking at its reinsurance purchases, especially if it is basically happy with the program it has. This would assume that the reinsurance is adding value, so could not be used to show that, but it could be used to calculate a risk-based profitability for each line that is consistent with the value function revealed by the reinsurance program.