Mortality Projection for US Workers Compensation Annuity Claims

Previous studies have found that mortality in the US for permanently injured workers after medical stabilization is similar to that for the population as a whole. This is higher mortality than for typical annuity recipients, who are usually select in some way, but it means that population mortality data is directly applicable. US male and female mortality trends are modeled using additive-multiplicative fixed effects and polynomial models for log-mortality, both adjusted in various ways for cohort effects. Particular trend histories for males and females under age 50 suggest that specific models be used for those ages, and patterns suggest that ages 90 and above have either data issues or a different trend before 1990. Thus the study is restricted to ages 50-89. The models are compared by penalized maximum likelihood. Somewhat different trends are found for male and female mortality.

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Studies such as ________ have found that permanently injured workers compensation claimants in the US after medical stabilization have mortality similar to the general population. Their injuries undoubtedly increase mortality risk, but they have guaranteed access to medical care, which seems to roughly compensate. Thus general population mortality data from the Human Mortality Database is used here. Several models of mortality trend patterns are fit and compared based on penalized maximum likelihood.

Years of death starting with 1971 were used, as 1970 was when the US reached its current geographic boundaries. There seems to have been a change in trend rates starting in 1975, so even though the models are fit before then, the trending begins from that point. Also preliminary data analysis found different trends for ages at death below 50, due perhaps to reproductive health issues and the impact of HIV during some of this period, so ages 50 and on were used. This was stopped at age 89, as older ages had quite unusual mortality patterns before 1990 – mortality reducing with age, etc. These could be data issues. The data available during this study ends with year of death 2005. All of this resulted in using year-of-birth cohorts 1882 to 1955.

Mortality data can be arranged quite similarly to loss development triangles. In casualty insurance the rows of the triangles are typically year of accident occurrence, and upward-right sloping diagonals are all in the same year of payment. The columns are called payment lags, but as any loss occurring in say 2007 and paid in 2008 is identified as lag 1, lag 1 can be anywhere from 0 to 2 years of time elapsed between occurrence and payment. In mortality studies the rows are typically age at latest birthday before death, and the columns year of death, so the similar diagonal is year of birth. This can be a two-year range, however, similar to lag in the casualty case.

Some of these choices are arbitrary, so to make the mortality triangle a little more like the casualty triangle, we take the rows to be year of birth and the columns age at death. Here the year of birth is implied by subtracting age at death from year of death, so is actually a two-year range, depending when in the year the birth and death were.

A popular model of mortality is Lee-Carter plus cohorts, from Renshaw-Habermann (2006) (RH). Using casualty-type notation for this, let d represent the age in a column and w the birth-year for a row. Then w+d is the year of death, so is constant on the diagonal. The mortality rate for a cell is $m_{w,d} = D_{w,d}/E_{w,d}$, where D and E are the deaths in the cell and the number alive from year of birth w at age d. This could be number alive at the beginning of the year but is more typically the average for the year. The log of the mean mortality rate for the RH model can be expressed as:

$$\log m_{w,d} = a_d + b_d h_{w+d} + c_d u_w$$

Without the last term this is the Lee-Carter model LC. Since a_d is usually close to the average of the log $m_{w,d}$, defining $x_{w,d} = \log m_{w,d} - a_d$ gives the model $x_{w,d} = b_d h_{w+d} + c_d u_w$. The RH model may be interesting for casualty insurance, taking x as losses in $x_{w,d} = b_d h_{w+d} + c_d u_w$. This makes the diagonal effect vary by lag, which could be reasonable. An intermediate model between RH and LC is to set $c_d = 1$. There are other possibilities too, such as making c_d piecewise linear, or only having it vary from 1 for a few ages. The LC model with $x_{w,d} = \log m_{w,d} - a_d = b_d h_{w+d}$ is a multiplicative cross-classified (fixed effects) model that can be fit by the classic Bailey-Simon (1960) iteration.

To interpret these models, a_d gives the shape of the mortality rate across ages. It is usually close to linearly increasing in d. Then h_{w+d} expresses the mortality trend. This is usually downward sloping as mortality has been decreasing at most ages, and is also somewhat linear, as there has been a fairly constant rate of decrease of log mortality. The b_d factor allows the mortality trend to differ by age. However each age has a constant differential in trend rate b_d in this model. This is where the LC model can run into differences from actual data, as some ages might trend faster or slower for a while but not always.

There is some ambiguity about the cohort effects u_w. It is not clear why some years of birth might have different changes in mortality than others. The earliest and latest years of birth in a typical study only appear in a few years of death, usually at the oldest and youngest ages, so the cohort parameters could essentially be picking up changes in the shape of the mortality curve rather than representing actual cohort differences that will persist outside of the dataset. The c_d factors allow the cohort effects to vary by age of death. This seems reasonable but again could allow the model to fit to changes in shape of the mortality rates that do not actually come from cohort effects.

There are some identifiability problems with these models. For instance, increasing every b by a factor and reducing every h by the same factor does not change the fitted values. It is similar for c and u. We will adopt the constraints that average(b_d) = average(c_d) = 1; $\Sigma_w u_w = h_{1975} = 0$. The trend is selected to start at 1975 as it appears to be downward and reasonably constant after that.

Given set of parameters, new parameters that meet the constraints can be produced using: $b_d^* = nb_d/\Sigma b_d$; $c_d^* = nc_d/\Sigma c_d$; $u_w^* = [u_w - \text{average}(u_w)]\Sigma c_d/n$; $h_k^* = [h_k - h_{1975}]\Sigma b_d/n$; $a_d^* = a_d + c_d\text{average}(u_w) + b_d h_{1975}$. Then $a_d^* + b_d^* h_{w+d}^* + c_d^* u_w^* = a_d + b_d h_{w+d} + c_d u_w$ and the constraints are met.

Another model, suggested by Cairns etal. _____ is the quadratic model. Log mortality is close to linear but not quite, so a quadratic model should be reasonable. (Cairns etal. use a slightly different transform than log before applying this model.) The starting point is to fit a quadratic curve to the log mortality rates for each year of death: $\log m_{w,d} = f_{w+d} + dg_{w+d} + d^2h_{w+d}$. A cohort effect can be added: $\log m_{w,d} = f_{w+d} + dg_{w+d} + d^2h_{w+d} + u_w$. The cohort effect could also vary by age: $\log m_{w,d} = f_{w+d} + dg_{w+d} + d^2h_{w+d} + r_d u_w$.

Also there is some indeterminacy in the quadratic model with cohorts. If you divide each r_d by a factor t, and multiply each u_w by the same t, then none of the values of $m_{w,d}$ are affected. But there is a more complicated interaction as well. If you divide each r_d by t and change each u_w to $u_w^* = t(u_w - a - bw - cw^2)$, for any values of a, b, and c, you can compensate and get the same fit if you also add $b+2c(w+d)$ to each g_{w+d}, add c to each h_{w+d}, and add $a+b(w+d)+c(w+d)^2$ to each f_{w+d}. The change in $\log m_{w,d}$ is then $a + b(w+d) + c(w^2+2dw+d^2) - bd - 2c(dw+d^2) + cd^2 - a - bw - cw^2 = 0$, so the fitted values are all the same.

Thus to make the parameters specific some constraints are needed. Suppose we have a set of estimated parameters and will apply constraints to get new parameters. One set of constraints is to first make the r_d average to unity by setting $t = \Sigma r_d/n$ and $r_d^* = r_d/t$. Then find the best fitting quadratic polynomial in w, $p(w) = a+bw+cw^2$, to the u_w, and subtract p(w) from each of the u_w and multiply by t. This makes the u_w parameters t times the residuals to the best fitting quadratic in w. Finally make the adjustments above to the other parameters.

To do the fit of p(w) by minimizing sum of squared errors, minimize $\Sigma_w(u_w - a - bw - cw^2)^2$, which gives the equations:

$\Sigma u_w = an + b\Sigma w + c\Sigma w^2$

$\Sigma wu_w = a\Sigma w + b\Sigma w^2 + c\Sigma w^3$

$\Sigma w^2u_w = a\Sigma w^2 + b\Sigma w^3 + c\Sigma w^4$

This gives 3 linear equations in the 3 variables a, b and c, which can be calculated directly to set $u_w^* = tu_w - tp(w)$. Then $f_{w+d}^* = a+b(w+d)+c(w+d)^2 + f_{w+d}$, and $g_{w+d}^* = b+2c(w+d) + g_{w+d}$, and finally $h_{w+d}^* = c + h_{w+d}$.

The quadratic model is not as flexible as LC in getting at the shape of the mortality curve in d, because it forces the shape to be a quadratic. But it allows more flexibility in changes in shape over time, which are quite limited in LC. The drawback to that is then you would have three parameters to project to get the future shapes of mortality curves, which could be difficult to do in a reasonable way for parameters of a quadratic.

To do MLE, E is considered deterministic and D stochastic. A typical assumption is that $D_{w,d}$ is Poisson in $m_{w,d}E_{w,d}$. The loglikelihood is then:

$$L = \Sigma_{w,d}\{D_{w,d}\log[m_{w,d}E_{w,d}] - m_{w,d}E_{w,d} - \log[D_{w,d}!]\} = \Sigma_{w,d}L_{w,d}$$

The RH and quadratic formulas for $m_{w,d}$ can be substituted here. To do the MLE, start with setting the partial derivatives of $L_{w,d}$ with respect to (wrt) each parameter to zero. The derivative of $L_{w,d}$ with respect to some parameter θ is $\partial L_{w,d}/\partial\theta = \partial L_{w,d}/\partial m_{w,d} * \partial m_{w,d} /\partial\theta$. It is easy to show that $\partial L_{w,d}/\partial m_{w,d} = R_{w,d}/m_{w,d}$, where $R_{w,d} = D_{w,d} - m_{w,d}E_{w,d}$ is the residual from the model. This holds for any of the models with the Poisson distribution.

In the quadratic model, as an example, get $\partial m_{w,d} /\partial\theta$ from $m_{w,d} = \exp(f_{w+d} + dg_{w+d} + d^2h_{w+d} + r_du_w)$ for the parameters θ either f_{w+d}, g_{w+d} , h_{w+d}, r_d or u_w. For instance, $\partial m_{w,d} /\partial g_k = dm_{w,d}$ if $w+d =k$ and 0 otherwise, and so $\partial L/\partial g_k = \Sigma_{w+d=k}\, dR_{w,d}$, etc.

For a particular j, differentiating the RH formula wrt a_j gives:

$\exp(a_j) = \Sigma_w D_{w,j} / \Sigma_w E_{w,j}\exp(b_jh_{w+j}+c_ju_w)$. Denote the modeled point as $\mu_{w,d} = E_{w,d}\exp(a_d+b_dh_{w+d}+c_du_w)$. The first and second derivatives of L wrt b_j are:

$\Sigma_w R_{w,j}h_{w+j}$; $\quad - \Sigma_w h_{w+j}^2\mu_{w,j}$

Wrt c_j are:

$\Sigma_w R_{w,j}u_w$; $\quad - \Sigma_w u_w^2\mu_{w,j}$

Wrt u_i are:

$\sum_d R_{i,d} c_d$; $\qquad - \sum_d c_d^2 \mu_{i,d}$

And wrt h_k are:

$\sum_{w+d=k} R_{w,d} b_d$; $\qquad - \sum_{w+d=k} b_d^2 \mu_{w,d}$

For the b, c, u and h parameters it is not possible to solve explicitly for the parameter in terms of the others and the data. If it were, you could iterate for the parameters in the fashion of Bailey and Simon (1960), i.e., solve for one set in terms of the others, going through each set in turn, and repeating until convergence. However Goodman (1979) provided a work-around in this case: do a Newton-Raphson iteration at each step instead of solving for the parameters exactly. Since you want the derivatives to be zero, the iteration for parameter θ is $\theta_{i+1} = \theta_i - L'/L''$, where the derivatives are wrt θ.

For starting parameters in the iteration, a_d is taken as the average over w of log $m_{w,d}$, each b_d and c_d is 1, and u_w and h_k start at zero. The latest estimate of the μs is used at each step. Since they are initially zero, the h and u parameters are iterated first, then b and c, then a, which has the best starting values. After all the parameters have been given a new iteration, the constraints can be applied to adjust the iterated parameters.

In the quadratic model with no cohort effects, for any year of death k you get three non-linear equations in three variables:

$\sum_{w+d=k} R_{w,d} = 0$; $\sum_{w+d=k} dR_{w,d} = 0$; $\sum_{w+d=k} d^2 R_{w,d} = 0$,

where in these sums $R_{w,d} = D_{w,d} - \exp(f_k + dg_k + d^2 h_k) E_{w,d}$

Since there are only three variables, any nonlinear minimizing routine can solve for them. In the more general case with the cohorts, the minimization is for a few hundred variables simultaneously, so an iteration like that for the RH model may be needed.

(show 1st and 2nd derivatives of quadratic model)

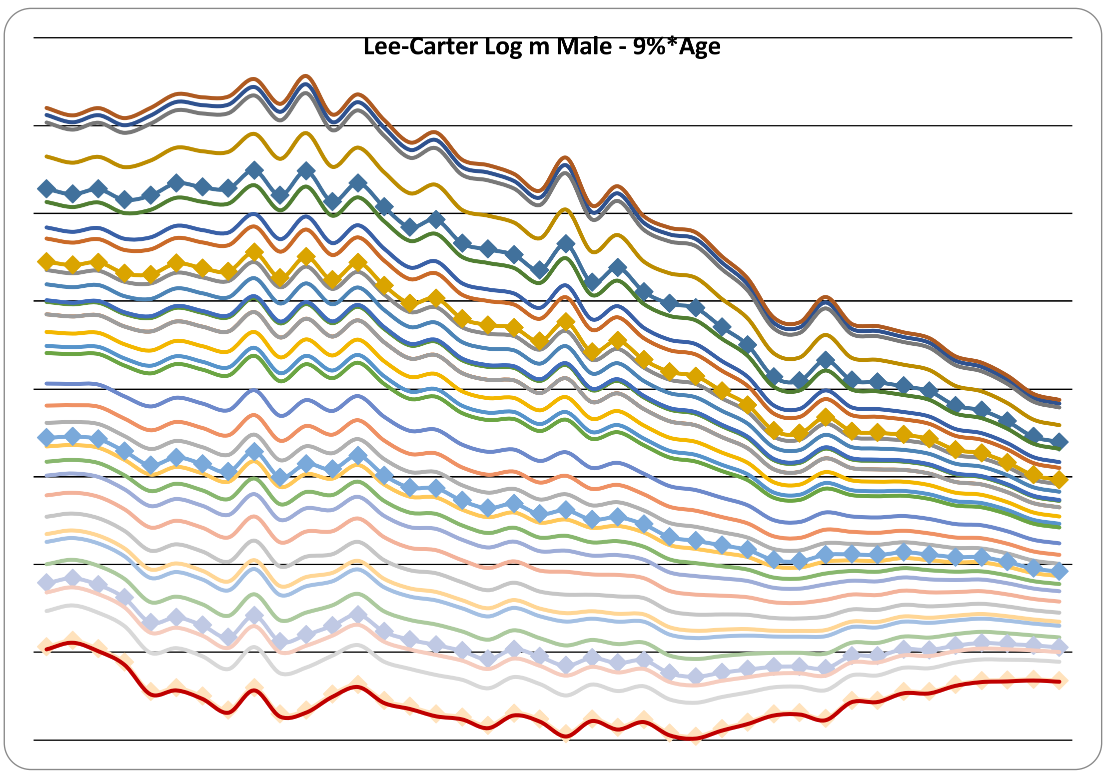
Fits

Goodness of fit of different models can be compared using penalized likelihood. The traditional comparison is to start with the negative loglikelihood, NLL, and subtract a penalty. We use the traditional criteria divided by 2, as these are more directly related to the NLL, but will use the standard names nonetheless. Thus the Akaike Information Criterion, AIC, uses a penalty of 1 for each parameter. If N is the sample size (number of observed cells), the Bayesian Information Criterion uses a penalty of ½ log N for each parameter. There is a growing consensus among information theorists that the AIC is too lenient on extra parameters, but the BIC is too punitive. The small sample AIC, or AICc, and the Hannan-Quinn Information Criterion, HQIC, are intermediate. The latter gives a penalty of log log N for each parameter. The AICc is a bit more complicated, increasing the penalty as the number of parameters increases.

LC and RH and the intermediate model $c_d = 1$ were fit to male and female mortality for the years described above. For both data sets, all the information criteria found that both RH and the intermediate model fit quite a bit better than LC, and that RH was a bit better than the intermediate model.

insert information criteria here

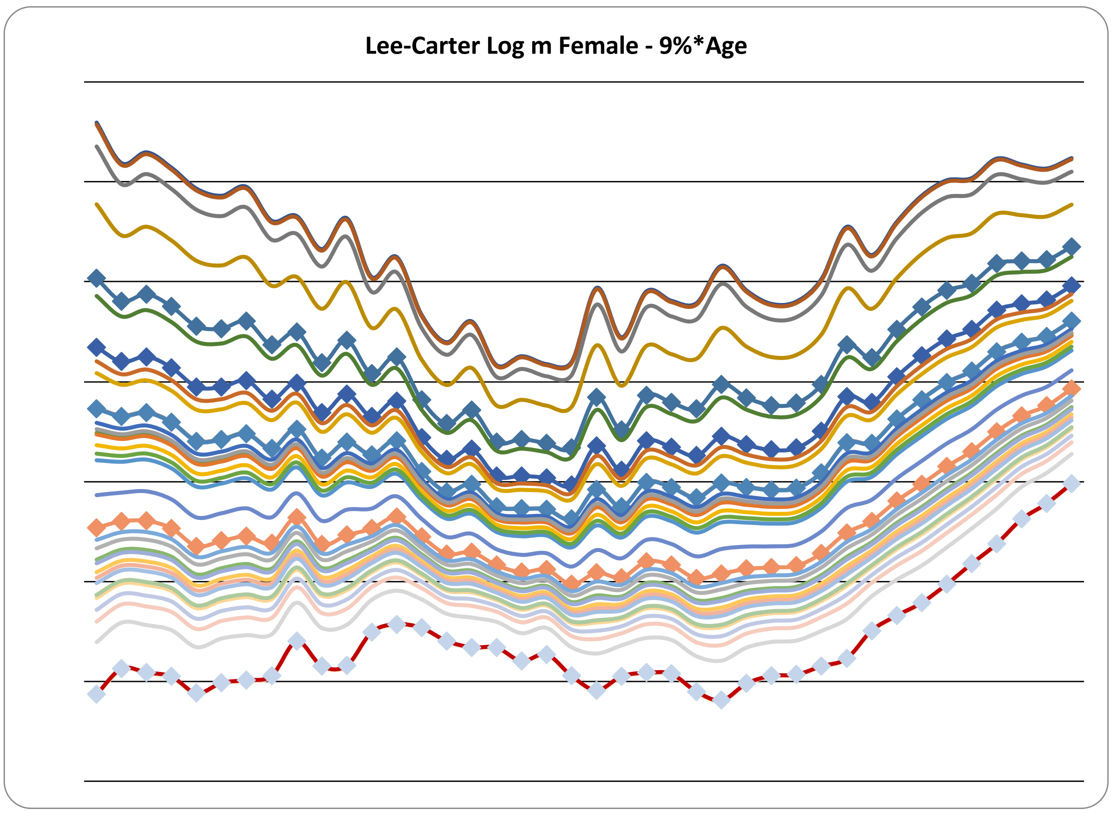
The LC fitted values are shown for males and females. What is graphed is log $m_{w,d} - 0.09d$ for both actual and fitted. This flattens out the graphs which allows for better visual comparisons.

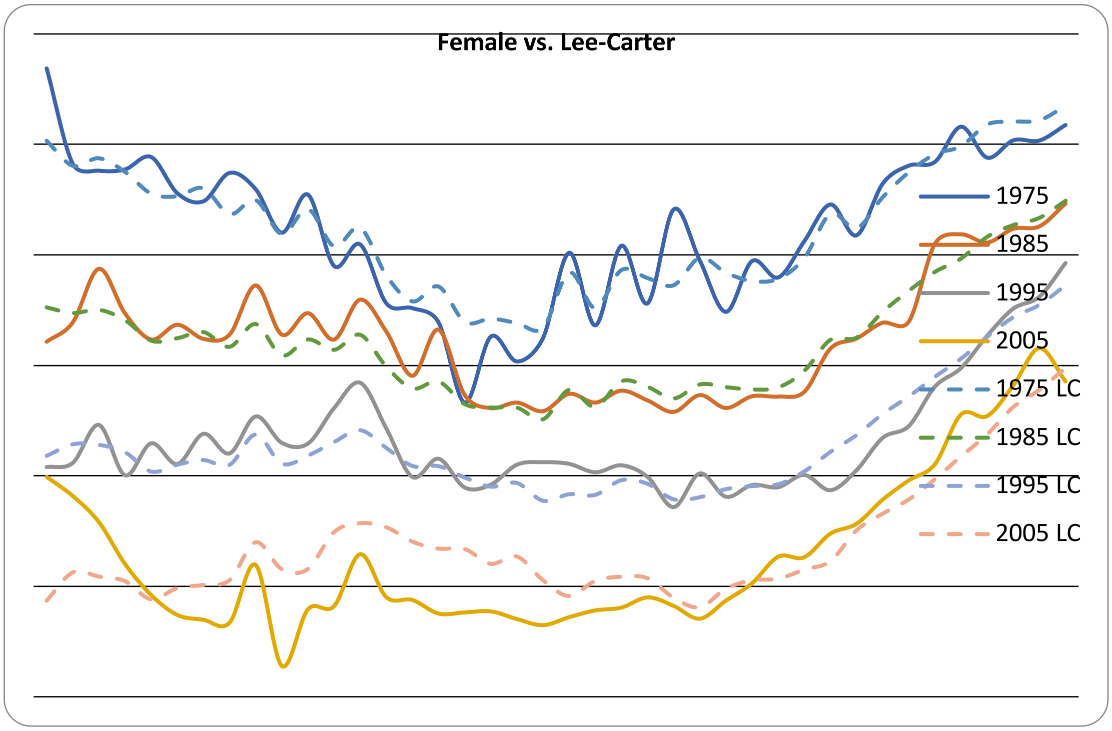


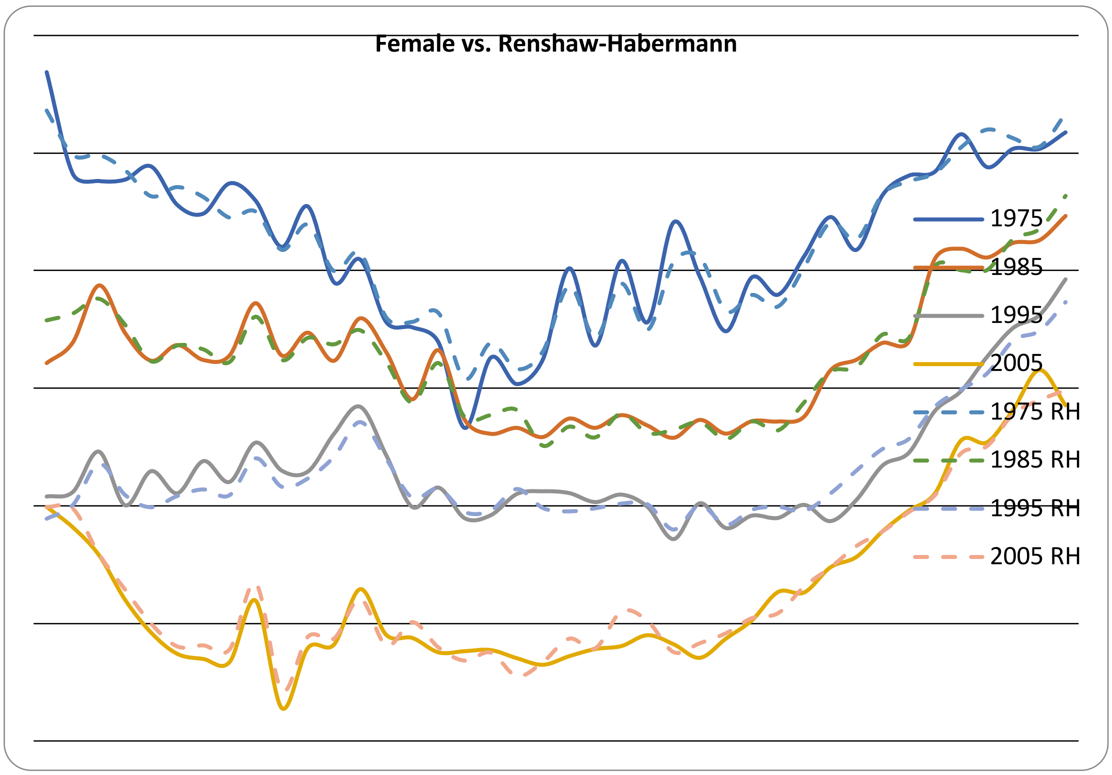
The more recent years are at the bottom of the graph, as that is where the mortality is the lowest. For males, the downward trend is less at the older ages. Years following periods of larger trend are shown with markers. These years are 1975, 1980, 1993, 2001, and 2004. The years 2004 and 2005 are almost identical.

For females the shape of the graph is somewhat different than for males, but there is not so much diminishing trend at older ages. The years following mortality improvements are 1975, 1977, 1981, 1990 and 2004. Again 2004 and 2005 are almost identical.

Graphs of the LC and RH models for years of death 1975, 1985, 1995, and 2005 are shown below to give a feeling for the fits.



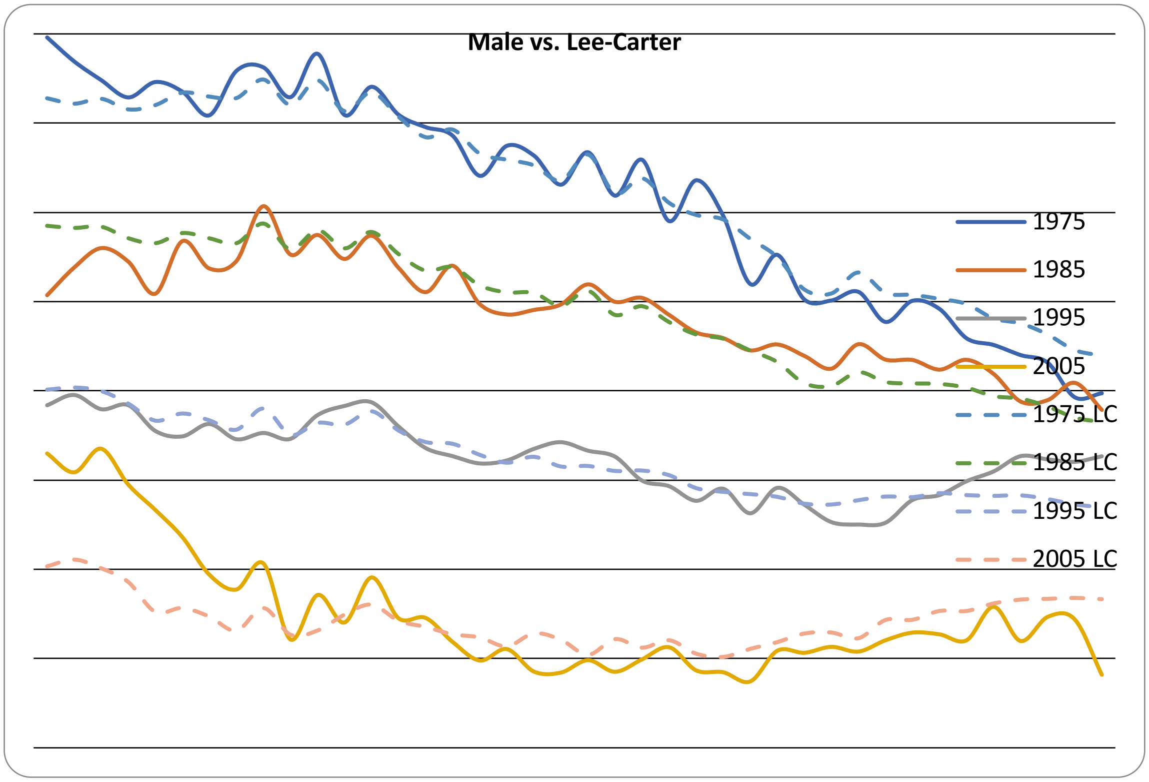


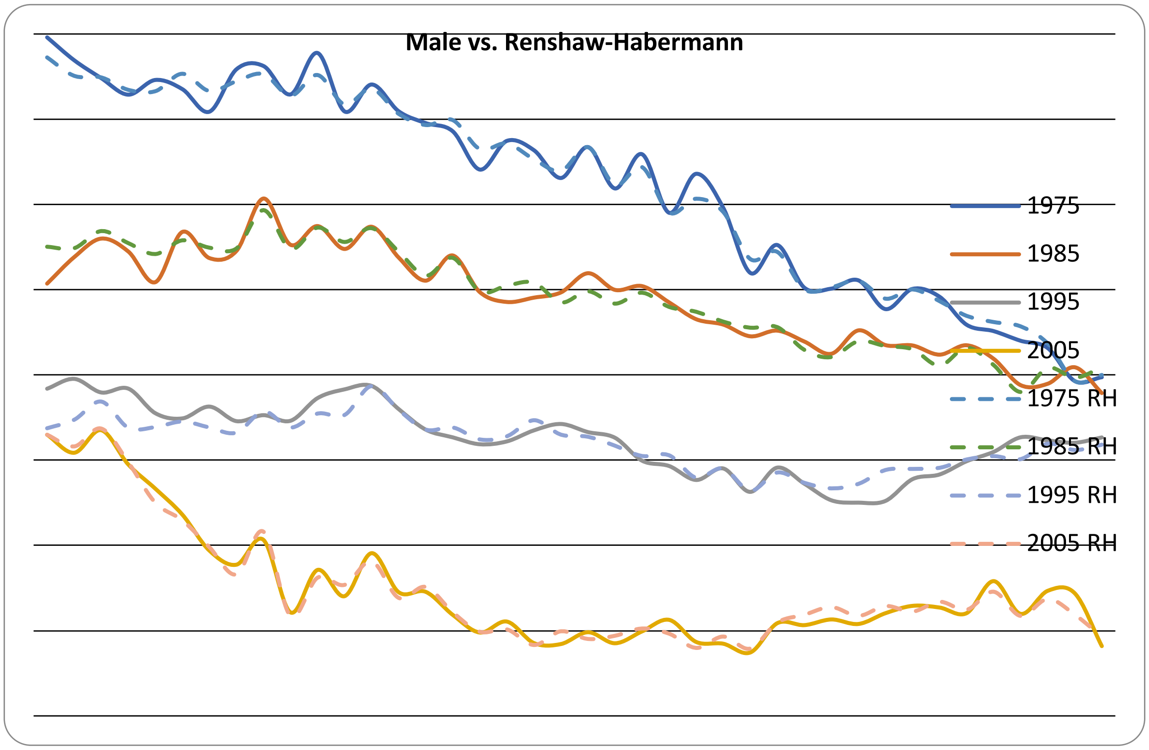


For the female data, the LC fits are close to parallel, but perhaps they are getting closer to each other on the right side, which is accomodated by the LC model. The fit is not nearly as good as for the RH model, but as discussed above that could be due to the RH model being able to use some cohort effects that only affect a few points and are not truly representative of those cohorts.

The male fits are similar. The LC model fits poorly at both ends of the lines, as there has been an apparent shift in the relative male mortality rates across ages. The RH model is able to correct for that, but again the interpretation that this is due to true cohort effects is suspect.

For both the male and female data in years 2000 to 2005, mortality rates for people in their early 50s does not have the downward trend that it does for earlier years, and which older ages continue to have. Only people born after 1946 have deaths in this range, so the RH model can make that a cohort effect for those birth years.





The Lee-Carter parameters are shown in Table 1. A model is a story of the data, and the story the Lee-Carter model is telling is that male mortality in 2005 is roughly comparable to female rates of 1972. Male rates have come down by 0.48 in the last 30 years, while female rates have decreased

Table 1 Lee Carter Parameters Female and Male

♀	Ad	Bd		Hw+d	♂	Ad	Bd		Hw+d
50	-5.40	1.44	**1971**	0.11	**50**	-4.77	1.09	**1971**	0.08
51	-5.33	1.28	**1972**	0.11	**51**	-4.69	1.07	**1972**	0.08
52	-5.23	1.31	**1973**	0.09	**52**	-4.59	1.10	**1973**	0.07
53	-5.15	1.29	**1974**	0.05	**53**	-4.51	1.10	**1974**	0.03
54	-5.08	1.27	**1975**	0	**54**	-4.42	1.18	**1975**	0
55	-5.00	1.23	**1976**	-0.01	**55**	-4.32	1.20	**1976**	-0.01
56	-4.90	1.25	**1977**	-0.05	**56**	-4.23	1.21	**1977**	-0.04
57	-4.83	1.15	**1978**	-0.06	**57**	-4.14	1.24	**1978**	-0.05
58	-4.73	1.08	**1979**	-0.09	**58**	-4.03	1.23	**1979**	-0.08
59	-4.67	1.05	**1980**	-0.07	**59**	-3.97	1.24	**1980**	-0.08
60	-4.56	1.13	**1981**	-0.09	**60**	-3.85	1.29	**1981**	-0.10
61	-4.50	0.90	**1982**	-0.11	**61**	-3.80	1.18	**1982**	-0.12
62	-4.40	0.93	**1983**	-0.10	**62**	-3.69	1.20	**1983**	-0.12
63	-4.35	0.79	**1984**	-0.11	**63**	-3.62	1.18	**1984**	-0.13
64	-4.28	0.75	**1985**	-0.10	**64**	-3.56	1.14	**1985**	-0.13
65	-4.18	0.82	**1986**	-0.12	**65**	-3.46	1.18	**1986**	-0.15
66	-4.12	0.71	**1987**	-0.13	**66**	-3.39	1.13	**1987**	-0.16

67	-4.03	0.77	**1988**	-0.12	**67**	-3.31	1.14	**1988**	-0.17
68	-3.94	0.73	**1989**	-0.15	**68**	-3.23	1.09	**1989**	-0.20
69	-3.86	0.79	**1990**	-0.17	**69**	-3.15	1.07	**1990**	-0.23
70	-3.72	1.02	**1991**	-0.19	**70**	-3.04	1.17	**1991**	-0.24
71	-3.66	0.86	**1992**	-0.20	**71**	-2.99	1.04	**1992**	-0.27
72	-3.53	0.96	**1993**	-0.18	**72**	-2.88	1.10	**1993**	-0.26
73	-3.45	0.94	**1994**	-0.19	**73**	-2.82	1.02	**1994**	-0.28
74	-3.37	0.98	**1995**	-0.20	**74**	-2.74	1.03	**1995**	-0.30
75	-3.25	1.10	**1996**	-0.21	**75**	-2.66	1.02	**1996**	-0.32
76	-3.18	0.99	**1997**	-0.22	**76**	-2.59	0.96	**1997**	-0.34
77	-3.09	0.94	**1998**	-0.22	**77**	-2.52	0.90	**1998**	-0.36
78	-3.00	0.94	**1999**	-0.21	**78**	-2.47	0.80	**1999**	-0.37
79	-2.89	0.98	**2000**	-0.22	**79**	-2.38	0.79	**2000**	-0.39
80	-2.76	1.09	**2001**	-0.23	**80**	-2.27	0.85	**2001**	-0.41
81	-2.69	0.95	**2002**	-0.24	**81**	-2.20	0.76	**2002**	-0.42
82	-2.57	0.99	**2003**	-0.25	**82**	-2.11	0.76	**2003**	-0.44
83	-2.46	1.03	**2004**	-0.29	**83**	-2.03	0.73	**2004**	-0.48
84	-2.35	1.02	**2005**	-0.29	**84**	-1.94	0.72	**2005**	-0.48
85	-2.25	0.97			**85**	-1.87	0.67		
86	-2.14	0.97			**86**	-1.78	0.65		
87	-2.05	0.90			**87**	-1.71	0.62		
88	-1.96	0.85			**88**	-1.63	0.58		
89	-1.86	0.82			**89**	-1.55	0.57		

by about 60% of that. The decrease in male rates is greatest in the younger years of the study – in this case the 50s. This shows up in the b_d parameters as well as the graphs. The trend has been less at older ages, especially for females. The improvement in mortality has been at a fairly steady rate since 1975, but there have been a few more dramatic jumps in some years. These years are a bit different for males and females. The actual improvement in mortality in the most recent years has been less than the model says for both males and females in the younger ages. For the older ages it has been more than the model for males but less than the model for females.

The RH parameters are in Table 2 except for the u_w parameters which are graphed below it. This model tells a somewhat different story. Male mortality rates for older ages in 2005 are similar to those for females in 1975, but for younger ages they are worse than females in 1975. Male and female log mortality rates have both come down by about 0.44 in the last 30 years. The downward mortality trend by age, as measured by b_d, has been greatest for older ages, especially for males. All of this is offset to some degree by the cohort effects, however. Females born before 1900 had substantially lower mortality, getting ever lower going backwards in time. This is somewhat the case for males as well, but to a much lesser extent. The highest mortality cohorts for males were births from about 1900 to 1920 and for females 1920 to 1940. Those born after those decades had improving mortality for a while, but 1948 was a turnaround year, and cohorts from that point on have had increasing mortality rates. The cohort effects for males are particularly strong at older ages.

The RH model fits the data much better but the story is less plausible. There is no clear explanation for the strong cohort effects, and since many cohorts affect only a few ages in a few years of death, they may not be true cohort effects that persist over ages outside of this data.

The LC story seems to correspond with the raw data. However it does have fitting problems, especially in recent years where the shape of the mortality curve appears to be changing. The shape shown in the graphs for 2005 actually begins to emerge around year 2000. There could be a number of reasons for the slowing of mortality improvement in the younger part of this group, including demographic changes and differential access to health care, as well as stabilization of the health improvements in some traditional illnesses such as heart disease and cancer for those age groups. The cohort effects from the RH model provide a much better fit but a less compelling story.

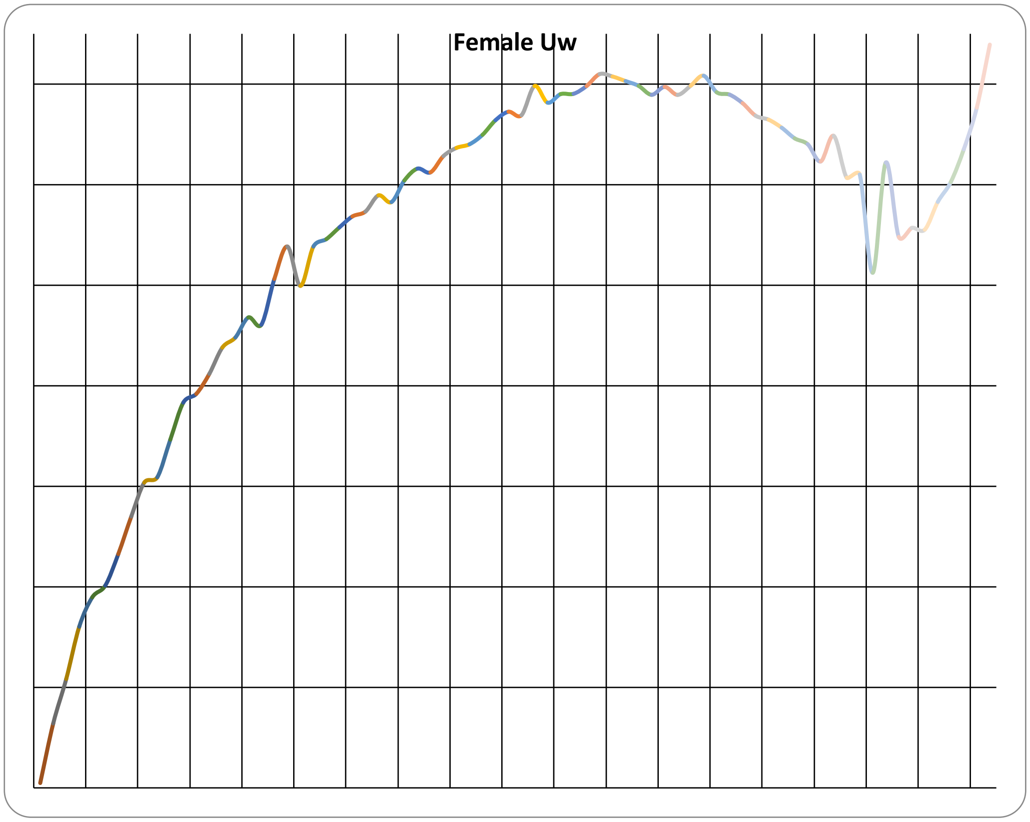
The changes in the shape of the mortality curve are probably more related to differential medical advances in diseases that affect different ages. The diseases that had been killing people in their early 50s may have already had major advances, so a recent slowing in the mortality trend for those ages is not surprising, but mortality is continuing to improve at later ages. Of course there could be changing demographics as well and data issues related perhaps to undocumented immigrants that could be affecting the reported numbers.

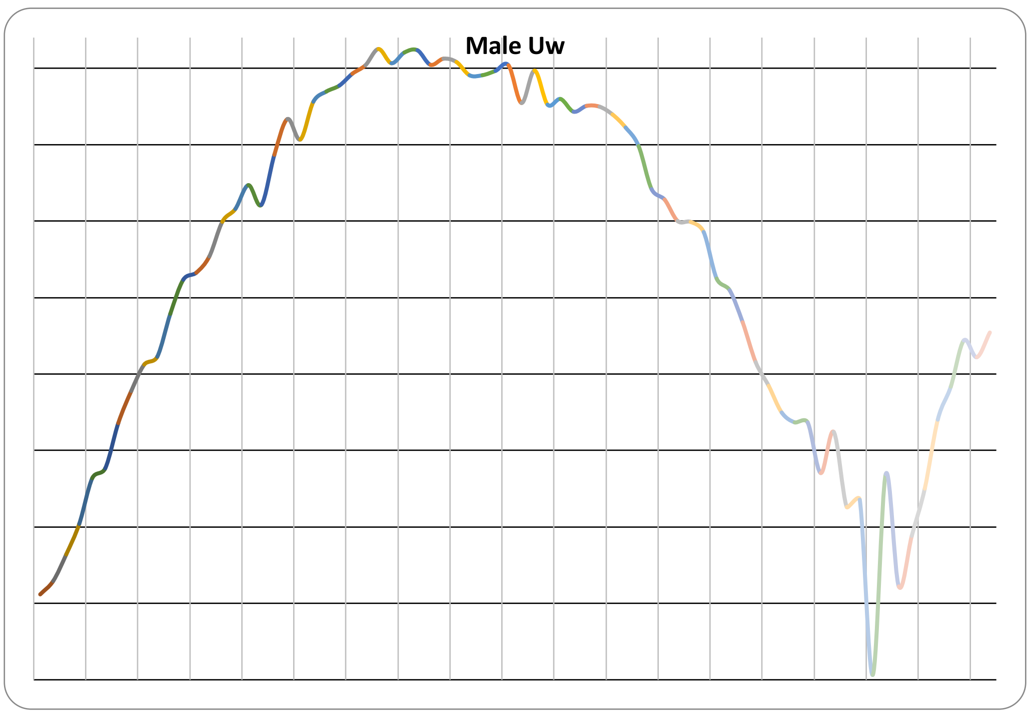
In any case, it appears that the LC model cannot accommodate the changes in the shape of the mortality curves that seem to be taking place. The RH model can, but its parameters say that male and female trends are about the same overall, and the greatest trends are for older ages, but that these are offset by strong cohort effects to produce the actual results. It seems more likely that the shape of the curves is changing, but neither of these models can reflect that.

The next step is to try the quadratic models. These give more crude approximations to the exact shape of the mortality curves, but allow for changes in shape over time, which may be revealing. However they are harder to trend than the LC family of models which only has one parameter trending over years of death.

Table 2 Renshaw Habermann Parameters Female and Male

♀	Ad	Bd	Cd		Hw+d	♂	Ad	Bd	Cd		Hw+d
50	-5.48	0.82	0.70	**1971**	0.16	**50**	0.16	0.77	0.71	**1971**	0.12
51	-5.44	0.65	0.86	**1972**	0.14	**51**	0.17	0.78	0.56	**1972**	0.12
52	-5.33	0.68	0.77	**1973**	0.11	**52**	0.15	0.68	0.74	**1973**	0.09
53	-5.24	0.69	0.67	**1974**	0.06	**53**	0.15	0.70	0.67	**1974**	0.05
54	-5.14	0.72	0.50	**1975**	0	**54**	0.17	0.81	0.62	**1975**	0
55	-5.08	0.67	0.69	**1976**	-0.02	**55**	0.17	0.79	0.64	**1976**	-0.02
56	-4.97	0.71	0.63	**1977**	-0.07	**56**	0.18	0.85	0.57	**1977**	-0.06
57	-4.89	0.69	0.55	**1978**	-0.09	**57**	0.21	0.97	0.45	**1978**	-0.07
58	-4.84	0.63	0.96	**1979**	-0.13	**58**	0.15	0.72	0.83	**1979**	-0.11
59	-4.74	0.66	0.65	**1980**	-0.12	**59**	0.20	0.93	0.51	**1980**	-0.11
60	-4.63	0.75	0.77	**1981**	-0.15	**60**	0.20	0.93	0.65	**1981**	-0.14
61	-4.60	0.64	1.08	**1982**	-0.18	**61**	0.17	0.78	0.75	**1982**	-0.16
62	-4.51	0.74	1.31	**1983**	-0.18	**62**	0.16	0.73	0.95	**1983**	-0.16
63	-4.45	0.67	1.24	**1984**	-0.19	**63**	0.17	0.81	0.80	**1984**	-0.18
64	-4.34	0.64	0.89	**1985**	-0.20	**64**	0.18	0.86	0.68	**1985**	-0.18
65	-4.25	0.77	1.18	**1986**	-0.22	**65**	0.18	0.85	0.84	**1986**	-0.20
66	-4.20	0.80	1.48	**1987**	-0.23	**66**	0.17	0.80	0.93	**1987**	-0.21
67	-4.08	0.82	1.23	**1988**	-0.24	**67**	0.19	0.89	0.82	**1988**	-0.22
68	-4.01	1.00	1.81	**1989**	-0.27	**68**	0.17	0.79	1.15	**1989**	-0.25
69	-3.88	0.97	1.39	**1990**	-0.29	**69**	0.18	0.86	0.99	**1990**	-0.28
70	-3.72	0.94	0.82	**1991**	-0.31	**70**	0.21	0.99	1.06	**1991**	-0.29
71	-3.66	1.13	1.54	**1992**	-0.33	**71**	0.19	0.90	1.09	**1992**	-0.31
72	-3.51	1.21	1.46	**1993**	-0.31	**72**	0.21	1.01	1.15	**1993**	-0.30
73	-3.42	1.12	1.21	**1994**	-0.33	**73**	0.21	0.97	1.26	**1994**	-0.32
74	-3.32	1.16	1.15	**1995**	-0.34	**74**	0.22	1.03	1.12	**1995**	-0.33
75	-3.19	1.20	1.00	**1996**	-0.35	**75**	0.23	1.07	0.93	**1996**	-0.34
76	-3.10	1.16	1.00	**1997**	-0.36	**76**	0.23	1.06	1.18	**1997**	-0.36
77	-2.99	1.26	1.19	**1998**	-0.37	**77**	0.22	1.05	1.12	**1998**	-0.37
78	-2.83	1.63	1.76	**1999**	-0.36	**78**	0.21	1.01	1.16	**1999**	-0.37
79	-2.72	1.53	1.46	**2000**	-0.37	**79**	0.22	1.01	1.02	**2000**	-0.38
80	-2.63	1.29	0.90	**2001**	-0.38	**80**	0.25	1.16	1.23	**2001**	-0.40
81	-2.55	1.19	0.84	**2002**	-0.39	**81**	0.24	1.12	1.22	**2002**	-0.40
82	-2.41	1.28	0.88	**2003**	-0.40	**82**	0.24	1.14	1.15	**2003**	-0.42
83	-2.28	1.32	0.85	**2004**	-0.44	**83**	0.25	1.17	1.18	**2004**	-0.45
84	-2.16	1.31	0.80	**2005**	-0.44	**84**	0.26	1.21	1.16	**2005**	-0.44
85	-2.07	1.23	0.70			**85**	0.27	1.29	1.31		
86	-1.98	1.13	0.55			**86**	0.29	1.34	1.33		
87	-1.85	1.20	0.66			**87**	0.32	1.49	1.53		
88	-1.68	1.40	0.85			**88**	0.36	1.68	1.79		
89	-1.49	1.62	1.02			**89**	0.43	2.00	2.19		


Female Uw


Male Uw

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Appendix 2 – Mixed 2nd Partials of RH

The derivative of the RH loglikelihood function wrt a_j is $\sum_w R_{w,j}$. The derivatives of that wrt any a_i, b_i or c_i with $i \neq j$ is zero. Its derivatives wrt a_j, b_j and c_j are $-\sum_w \mu_{w,j}$, $-\sum_w h_{w+j}\mu_{w,j}$, and $-\sum_w u_w\mu_{w,j}$, respectively, so these are the corresponding 2nd partials for a_j. For u_i the partial is $-c_j\mu_{i,j}$ and for h_k it is $-b_j\mu_{k-j,j}$, so these are the corresponding 2nd partials with a_j.

For b_j, the derivative of the loglikelihood is $\sum_w R_{w,j}h_{w+j}$. Thus the mixed 2nd partial wrt b_j and c_j is $-\sum_w h_{w+j}u_w\mu_{w,j}$. For b_j and u_i the 2nd partial is $-h_{i+j}c_j\mu_{i,j}$. For b_j and h_k it is $R_{k-j,j} - b_jh_k\mu_{k-j,j}$.

For c_j, the derivative of the loglikelihood is $\sum_w R_{w,j}u_w$. For c_j and h_k the 2nd partial is $-b_ju_{k-j}\mu_{k-j,j}$. For c_j and u_i it is $R_{i,j} - c_ju_i\mu_{i,j}$.

For u_i, the derivative of the loglikelihood is $\sum_d R_{i,d}c_d$. For u_i and h_k the 2nd partial is $-b_{k-i}c_{k-i}\mu_{i,k-i}$ but this is only for k up to i+50.