Parameter Risk Curriculum

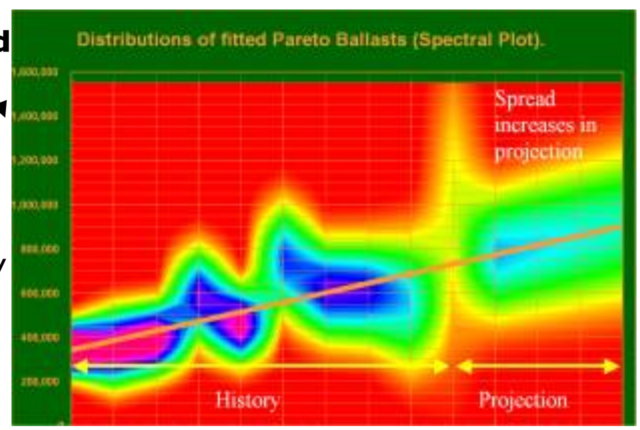
Topics:
Types of parameter risk
Discussion of each type
Impact of projection risk: development of formula through collective risk theory, and examples showing impact on large vs. small companies. Easy to do: formulas are simple and easy to apply.
Mathematical development of estimation risk calculations in 3 cases: large sample severity, Bayesian frequency, and small sample severity using Rodney's Bayesian approach to testing.

Slide summaries:

Parameter Risk
Includes estimation risk, projection risk, and event risk
These are systematic risks – do not reduce by adding volume
For large companies parameter risk could be the largest risk element
Comparable to cat risk before reinsurance
Greater than cat risk after reinsurance

Projection Risk
Change in risk conditions from recent past
In part due to uncertain trend
Can include change in exposures
More driving as gas prices change and other transportation looks risky
New types of fraud become more prevalent

Projection Risk from Uncertain Trend



Estimation Risk
Never enough data to know true parameters
More data and better fits reduce this risk – but never gone
Statistical methods can quantify this uncertainty

Parameter Risk – "Events" – Environmental changes caused by:
Government/Regulators
Consumer groups
Legal climate
"Hidden" peril
Competitors
Rating agencies
Historical data cannot reflect
Recognize this risk through:
Increased parameter risk
Judgmental adjustment of data and/or parameters

Mathematical Development Topics
Quantifying impact of projection risk
Estimation risk 1 – asymptotic theory
Estimation risk 2 – Bayesian updating
Estimation risk 3 – both together

Impact of Projection Risk

Collective risk theory

$L = \sum_{i=1}^{N} X_i$

$E(L) = E(N)E(X)$

$Var(L) = E(N)Var(X) + E(X)^2 Var(N)$

$CV(L)^2 = Var(X)/[E(N)E(X)^2] + Var(N)/E(N)^2$

$= [CV(X)^2 + VM(N)] / E(N)$

$= [\ 49 + 1\ ] / E(N)$

Goes to zero as frequency gets large

Add projection risk

$K = JL$, $E(J) = 1$, so $E(K) = E(L)$

$CV(K)^2 = [1+CV(J)^2]CV(L)^2 + CV(J)^2$

Lower limit is $CV(J)^2$

Impact of Projection Risk **CV(X)=7, VM(N)=1, CV(J)=0.05**

CV(J) E(N):	2,000	20,000	200,000
0.05	16.6%	7.1%	5.2%
0.03	16.1%	5.8%	3.4%
0.01	15.8%	5.1%	1.9%
0.00	15.8%	5.0%	1.6%

Effect on Loss Ratio **E(LR)=65, Uncertainty CV = 0.05, Lognormal Distribution**

Uncertainty	Small	Medium	Large
90th	79.2	71.0	69.4
95th	84.1	72.8	70.8
99th	94.1	76.4	73.3
No Uncertainty			
90th	78.5	69.2	66.3
95th	83.1	70.5	66.7
99th	92.5	72.9	67.4

Loss ratios in red illustrate 99th percentile with and without parameter uncertainty. Look at impact on a stop loss with retention 70%.

Estimation Risk I – Asymptotic Theory

At its maximum, all partial derivatives of log-likelihood function with respect to parameters are zero

2nd partials are negative

Matrix of negative of expected value of 2nd partials called "information matrix"

Usually evaluated by plugging maximizing values into formulas for 2nd partials

Matrix inverse of this is covariance matrix of parameters

Distribution of parameters is asymptotically multivariate normal with this covariance

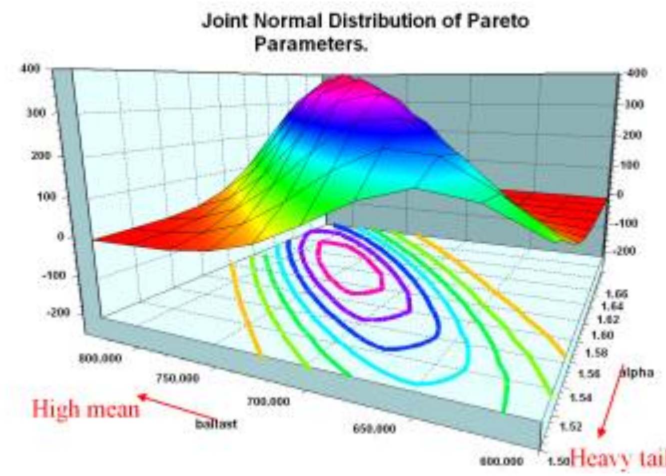
Estimation Risk I – Asymptotic Theory

See Loss Models p. 63 for discussion

Simulation proceeds by simulating parameters from normal, then simulating losses from the parameters

See Loss Models p. 613 for how to simulate multivariate normals

Information Matrix – Pareto Examp



Estimation Risk 2 – Bayesian Updat

Bayes Theorem:

$f(x|y)f(y) = f(x,y) = f(y|x)f(x)$

$f(x|y) = f(y|x)f(x)/f(y)$

$f(x|y) \propto f(y|x)f(x)$

Diffuse priors

$f(x) \propto 1$

$f(x) \propto 1/x$

Estimation Risk 2 – Bayesian Updating - Frequency Example

Poisson distribution with diffuse prior $f(\lambda) \propto 1/\lambda$

$f(k|\lambda) = e^{-\lambda}\lambda^k/k!$

$f(\lambda|k) \propto f(k|\lambda)/\lambda \propto e^{-\lambda}\lambda^{k-1}$

Gamma distribution in k, 1

That's posterior; predictive distribution of next observation is mixture of Poisson by that gamma, which is negative binomial

After n years of observation, predictive distribution is negative binomial with same mean as sample and variance = mean*(1+1/n)

Parameter distribution for λ is gamma with mean of sample and variance = mean/n

Can simulate from negative binomial or from gamma then Poisson

Estimation Risk 3 – Bayes + Information

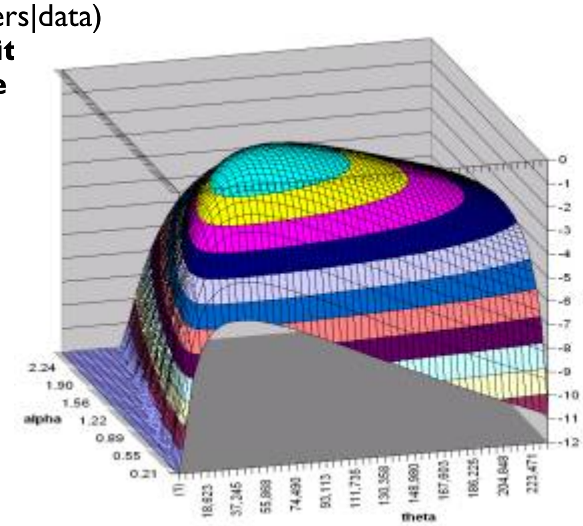
Tests developed by Rodney Kreps

Likelihood function is proportional to f(data|parameters)

Assume prior for parameters is proportional to 1

Then likelihood function is proportional to f(parameters|data)

Likelihood Function for Small Sample Pareto Fit
Scaled to Max = 1, Log Scale

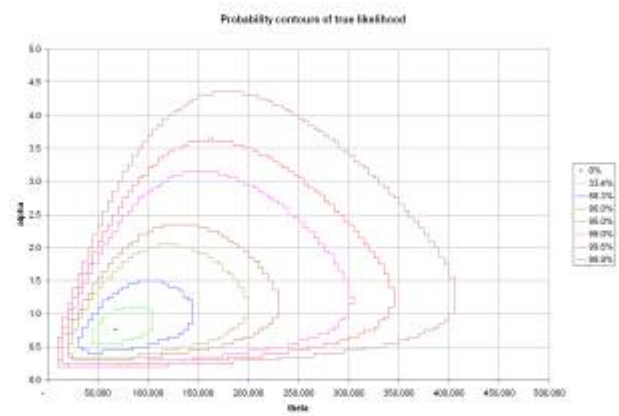


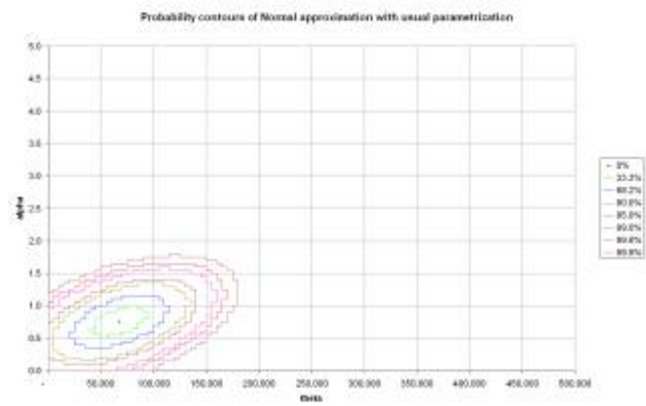
Testing Assumptions of Parameter Distribution
Use information matrix to get covariance matrix of parameters
This is quadratic term of expansion of likelihood function around max
Compare bivariate normal and bivariate lognormal parameter distributions to scaled likelihood function

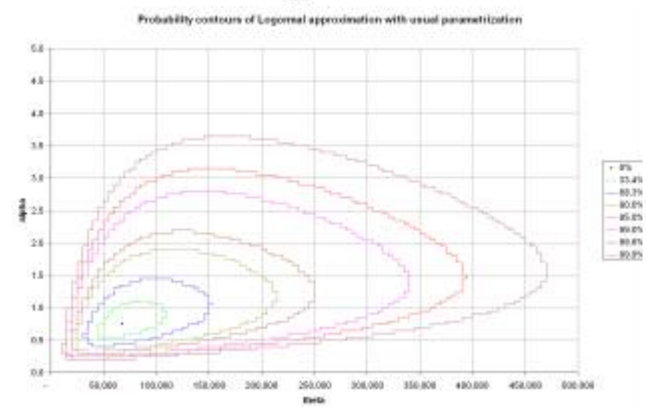
Probability Contours of Likelihood This and next four are Rodney's pictures

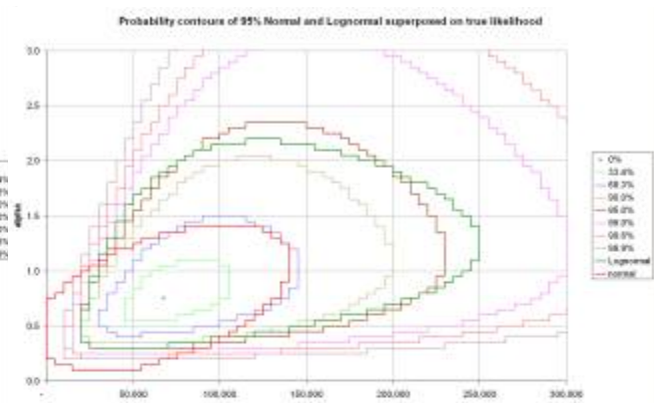
Normal Approximation, Lognormal Approximation Contours

95th Percentile Comparison, 33rd Percentile Comparison









Parameter Risk Summary
One of the largest elements of under-writing risk
Can quantify projection risk and estimation risk statistically and event risk by professional judgment
Included in Metarisk