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Correlation Effects on Loss Reserve Ranges

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Abstrat Correlation among lines of business can be built into reserve estimates or it can be added in later when calculating ranges. This note looks at the latter approach in the context of a regression model that includes row, column, and diagonal effects,

Correlation Effects on Loss Reserve Ranges

There is a growing recognition among actuaries that loss development may be correlated across lines of insurance. For instance Brehm (2002) discusses using correlation to project reserve ranges with correlated lines all estimated by a model discussed by Zehnwirth (1994). Taylor () proposed using a generalized linear model to simultaneously estimate reserves for correlated lines. Gillet and Serra (2003) do this using copulas under two correlation structures: the same cell is correlated across lines, or the entire development year is correlated across lines.

This note assumes losses including reserve ranges and parameter uncertainty have been estimated separately for each line, and then looks at how correlation could be used to estimate reserve ranges for their sum. It starts with the regression model from Brehm's paper.

This models the incremental paid losses in the *i,j* cell (accident year *i*, development year *j* and so calendar year *i+j*) as an accident year level parameter times the cumulative product of development year and calendar year annual effects times a lognormal error term. Taking logs converts this to a sum of effects. Thus denoting the ln of the incremental loss as y_{ij}, the model can be expressed as:

$$y_{ij} = \alpha_i + \sum_{k=1}^{j} \gamma_k + \sum_{t=1}^{i+j} \iota_t + \varepsilon_{ij}$$

Thus $\exp(\alpha_i)$ is the accident year level factor, the development year factor is the product of the decay elements $\exp(\gamma_k)$ for all development years up to *j*, all the trends $\exp(\iota_t)$ are multiplied for all calendar years up to *i+j*, and the error term is normal. Note that this is not a development model, as the previously emerged losses do not enter.

This model was fit to 10 years of US industry Schedule P paid loss ratios for private passenger auto liability, commercial auto liability, and other liability using Zehnwirth's software. These are sample fits, not necessarily optimized. The parameter estimates and their standard errors are shown in Exhibit 1.

Exhibit 1 – Triangle Model Parameters and SE's

	PPAL	SE	CAL	SE	OL	SE
			Gammas			
0-1	-0.235	2.2%	0.128	3.1%	0.153	2.2%
1-2	-0.797	2.3%	-0.302	1.6%	0.153	2.2%
2-3	-0.643	1.3%	-0.302	1.6%	-0.235	4.2%
3-4	-0.643	1.3%	-0.640	1.3%	-0.361	1.8%
4-5	-0.755	1.1%	-0.640	1.3%	-0.361	1.8%
5-6	-0.755	1.1%	-0.640	1.3%	-0.361	1.8%
6-7	-0.755	1.1%	-0.937	5.2%	-0.584	6.2%
7-8	-0.597	2.4%	-0.502	4.1%	-0.201	5.0%
8-9	-0.597	2.4%	-0.502	4.1%	-0.201	5.0%
			Alphas			
1993	-1.241	1.8%	-1.715	2.4%	-2.479	4.9%
1994	-1.241	1.8%	-1.715	2.4%	-2.479	4.9%
1995	-1.241	1.8%	-1.715	2.4%	-2.479	4.9%
1996	-1.241	1.8%	-1.715	2.4%	-2.479	4.9%
1997	-1.311	2.1%	-1.715	2.4%	-2.479	4.9%
1998	-1.311	2.1%	-1.715	2.4%	-2.479	4.9%
1999	-1.276	2.4%	-1.715	2.4%	-2.479	4.9%
2000	-1.276	2.4%	-1.715	2.4%	-2.479	4.9%
2001	-1.276	2.4%	-1.959	4.7%	-2.479	4.9%
2002	-1.276	2.4%	-1.959	4.7%	-2.479	4.9%
			Iotas			
1993-1994	0%	0%	0%	0%	3.9%	1.3%
1994-1995	0%	0%	0%	0%	3.9%	1.3%
1995-1996	0%	0%	0%	0%	3.9%	1.3%
1996-1997	0%	0%	0%	0%	3.9%	1.3%
1997-1998	0%	0%	0%	0%	3.9%	1.3%
1998-1999	0%	0%	8.0%	1.2%	9.5%	1.2%
1999-2000	9.0%	2.1%	8.0%	1.2%	9.5%	1.2%
2000-2001	0%	0%	0%	0%	9.5%	1.2%
2001-2002	0%	0%	0%	0%	0%	0%

Since these were loss ratio triangles, there is not a trend present across accident years (alphas) like there might be for losses. A change in alpha is related to rate adequacy changing, which could be unanticipated trends or competitive pressures, for example. A calendar year trend would increase later payments on an accident year relative to earlier ones and so if consistent would be hard to disentangle with the payment decays over time, represented by the gammas. Also it would not be necessary to do this, as the combination is what goes into the model results. It is the changes in iota that create unique effects. These could be driven by price trends in the economy, so it is not surprising to see them move together for different lines.

Several correlations were present in the fitting. Some are presented in Exhibit 2.

Exhibit 2 - Correlation			
	PPAL-CAL	**PPAL-OL**	**CA-OL**
Residuals	11.2%	34.3%	9.7%
Gammas	74.8%	51.0%	89.9%
Iotas	66.1%	46.5%	70.2%

For the purpose of imputing a dependence structure to the parameters of the model, three t-copulas were introduced, one for gammas, one for alphas, and one for iotas. The t-copula parameters consist of a correlation matrix for all the lines (3x3 in this case, so 3 unique non-diagonal parameters) and a single tail parameter that determines the degree to which the correlations continue into the tail. As that parameter increases, the t-copula approaches the normal copula, for which the extreme tails of the distributions are uncorrelated.

An estimation procedure for the 12 copula parameters was devised along the lines of the simulated method of moments (see, for example, Duffie and Singleton (1993)). The dependency structure of the three original y_{ij} triangles was described by a vector f_0 of correlation coefficients. Correlations were computed for each corresponding row, column, and diagonal for each pair of lines. Thus there were 3 sets of correlations for the first row, 3 for the first column, etc. This gave a set of 81 correlations, as the 10^{th} row, column, and diagonal each had only a single element so could not be correlated among triangles. Then for a trial set of the 12 copula parameters, 500 instances of the three triangles were simulated, and the correlation vector f_{sim} was computed for the simulated triangles and averaged over the simulations. The 12 copula parameters that minimize the expected distance between f_0 and f_{sim} were sought. A global genetic algorithm was used for this optimization (see the Appendix).

The simulation proceeded as follows. Each line of business has 9 gamma parameters. These were assumed to be normally distributed with means and standard deviations from Exhibit 1. The gammas (development) from 0 to 1 were simulated jointly with the dependency structure determined by the t-copula for gammas, and the marginals from the normal distributions based on Exhibit 1. This was repeated for each of the other lags, giving 27 gammas that are independent across lags but correlated across lines. This was done for alphas using the alpha t-copula parameters and for iotas using the iota t-copula parameters. Then error terms were drawn for each cell using the correlation of the residuals and the standard deviations of the residuals for each triangle, based on a normal copula

References

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