Risk Management Tests of Yield Curve Scenario Generators

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Abstract

Stochastic yield curve generators are often evaluated on how well they price options, and so there is a slant towards risk-neutral conditions. For risk management, however, real-world yield curves are needed and the set of scenarios should have realistic properties. Historical observations are the best gauge of what is realistic, but there is a tension between long-term properties and what is likely to prevail in the current environment, and this has to be recognized throughout the testing process. Tests from several sources are gathered and applied to a few generators of US and UK risky and risk-free bond yield curves as illustration.

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1 Introduction

A good deal of research in financial markets has been motivated by making money through identifying price opportunities. A growing emphasis recently is on the less ambitious goal of at least understanding the risks of current positions. Stochastic generators of yield curves used for risk management purposes produce scenarios of yield curves whose distribution is intended to be real-world, not risk-neutral. Various historical properties of past yield curves can be used to evaluate the distribution of scenarios in this context.

When fitting models to data, there is a wealth of statistical-inference tools to apply to test the appropriateness of the model. For stochastic generators, however, the bottom line is how well they forecast distributions of outcomes. In some cases in fact the models are calibrated to current options prices, so there is no parameterization with historical data to analyze. The generators might even be black boxes as far as the users are concerned. But all the generators produce scenarios that represent distributions of possible outcomes, so testing can focus on how realistic these distributions are.

Section 2 reviews some of the suggested tests in the literature and stylizes some facts to create a set of scenario

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tests. Section 3 introduces two stochastic generators which will serve as test dummies. The various tests are then applied to the trial generators in Section 4 for US rates and in Section 5 for UK. Section 6 concludes.

2 Proposed Tests

2.1 Some historical tests

Cairns (2004) and Backus et al. (1999) looked at statistical properties of interest rates, such as how means and volatilities vary by maturity, as well as correlations of yields across maturities and autocorrelation of the rates. Venter (2004) quantified yield-curve shapes as pairs of values, where the first value is the three-month rate and the second is a yield spread, such as the 10-year − 3-year spread or the 30-year − 10-year spread. The pairs can be plotted in two dimensions, and display specific statistical properties.

Interest rates display remarkably long auto-correlations, but the tests illustrated here are restricted to a single year projection, so this element is not emphasized. Correlations among risky and risk-free rates and various maturities will be reviewed. Tests based on moments are outlined next, then the two-dimensional yield-shape tests are presented, followed by a discussion of related tests of curve shapes.

2.2 Moment-based tests

In going from shorter to longer maturities, mean interest rates tend to increase, while their volatilities tend to decrease. This holds for both risky and risk-free rates, as well as for credit spreads, although at the longest ages the means might no longer increase and can even decline slightly. Short and long rate movements are highly correlated, as are risky and risk-free rates. Risk-free rates and credit spreads may or may not be correlated.

Almost all the generators produce upward-sloping yield curves most of the time, but downward slopes are possible occasionally, so overall slope will not be used as a test criterion. Credit spreads also tend to increase by maturity, as can be seen in Table 1. This will be used as a test, even though the actual spreads at any point may differ from the long-term average.

Table 1: Average spreads for US industrial bonds by maturity 1995–2011

Maturity	0.25	1	3	10	30
AA	0.55%	0.58%	0.69%	0.79%	1.18%
BBB	1.23%	1.25%	1.43%	1.58%	2.09%

Table 2 shows the standard deviations of yields for the period 1995 to 2011 for US Treasury and Industrial AA and BBB bonds. For all the bonds, the longer yields are more stable. For the shorter maturities, the riskier bonds are more stable, whereas the opposite is the case for the longer maturities. Conditionally given the starting yield curves, these same volatilities may not occur, but the increase in stability with maturity would be anticipated.

Table 2: US standard deviation by maturity 1995–2011

Maturity	0.25	1	3	10	30
Treasury	1.44%	1.35%	1.08%	0.69%	0.59%
AA Industrial	1.39%	1.34%	1.11%	0.78%	0.62%
BBB Industrial	1.33%	1.33%	1.12%	0.90%	0.72%

Bond yields are all strongly correlated, even though the correlation of spreads with rates changes with other economic conditions. The higher-rated bonds show more correlation with risk-free rates, as do the shorter maturities. One driver of those higher correlations is that their spreads are lower. Another consideration is that the

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larger spreads may be more negatively correlated with risk-free rates. It would seem reasonable to look for correlations of rates across generated scenarios to bear some similarity to historical patterns.

Table 3: Correlations with Treasuries for US industrial bond yields by maturity 1995–2011

Maturity	0.25	1	3	10	30
AA	97%	99%	98%	96%	88%
BBB	92%	94%	91%	83%	75%

2.3 Curve-shape tests

The yield curve using common maturities can be expressed as a vector. There are various ways to represent essential aspects of the vector in lower dimensions, such as yield spreads. Venter (2004) used pairs [3-month rate, yield spread] and found that these pairs tend to fall in certain ranges of the two-plane. This can also be expressed as a regression of the spreads on the short-rates.

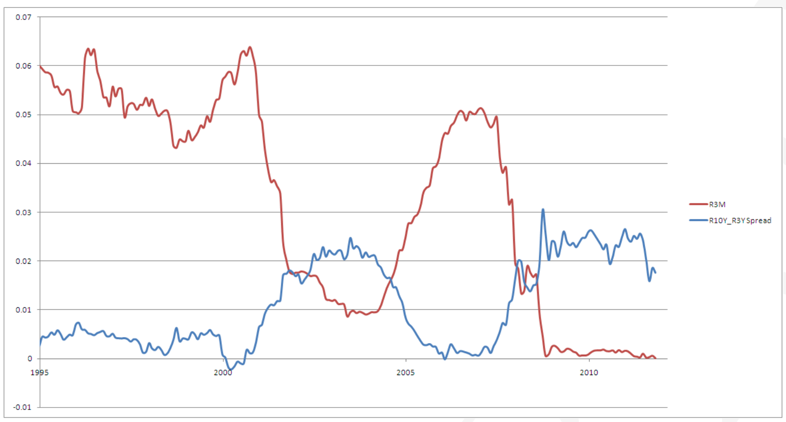

Figure 1: US Treasury ten-year – three-year spread and three-month rate

Here we look at the 10-year – 3-year spread, the 30-year – 10 year spread, and the slope change between the two spreads, all as related to the 3-month rate. The slope change is the 10 – 3 spread divided by 7 less the 30 – 10 spread divided by 20, and will be referred to here as the twist. Curvature is similar to twist. It calculates the slope change for all maturities. For example, the curvature at 5 year is the 5 – 4 spread divided by 1 less the 6 – 5 spread divided by 1.

Figure 1 is a graph of the 10 – 3 spread and the three-month rate since 1982. The two series clearly move in opposition to each other. Figure 2 shows a regression line through the pairs with the pairs color-coded by time period. The rates were different in different periods, but the dispersion around the regression line is pretty similar in each period. The line and the dispersion thus represent aspects of the distribution of yield-curve shapes that has held over time.

The comparable graphs for the 30 – 10 spread and the twist look very similar to these if separate scales are used for these items, as they are smaller in magnitude. Testing will compare the slope of the fitted lines and the dispersion around the lines for history and the scenarios generated by the models. The historical empirical values of these quantities are shown in the testing section.

Some commentators have applied curvature tests for forward rates at each point in the yield curve. These are similar to the twist test but at every maturity and these will also be run.

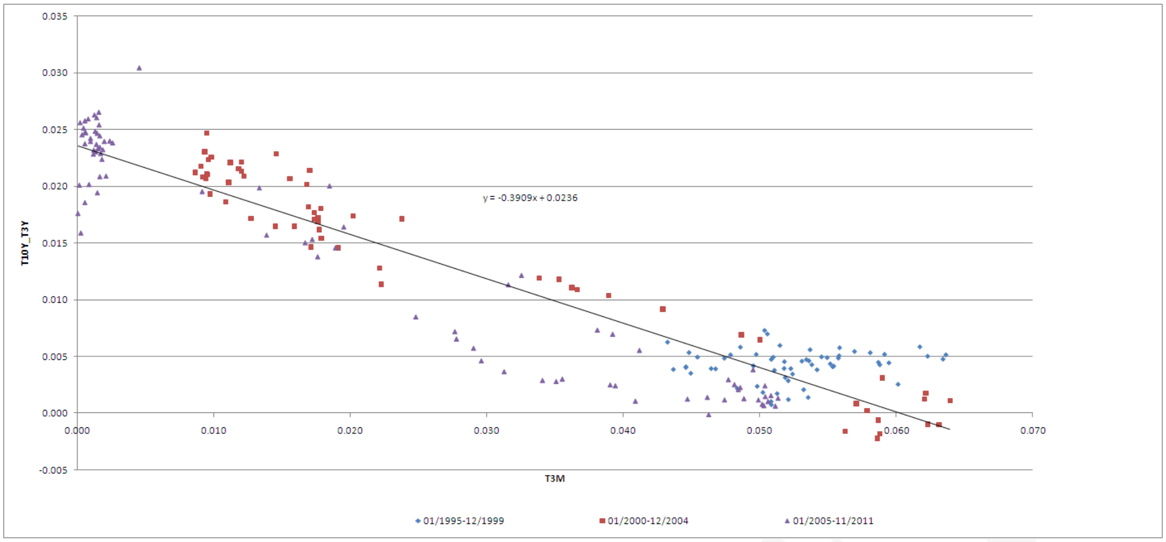

Figure 2: US Treasury ten-year – three-year spread vs. three-month rate and regression line 1995 - 2011

3 Models

Two of the models we test are short-rate models, a type of model that focuses on the dynamics of the short-term rate and build up yield curves from the projected behavior of the short-rate process. The third is the London Market Model, or LMM, which models the entire forward-rate curve and its dynamics over time.

3.1 Model A

Two models we test are from a third-party vendor. The third, Model A, is a short-rate model fit internally to have a base for reference. There is an extensive literature on short-rate models. For instance, Venter (2004) tests the stochastic-volatility model of Andersen and Lund (1997), which has a geometric Brownian motion for one volatility factor, and this is then multiplied by a power (itself a parameter) of the interest rate to get the volatility of the interest process. This is one of the better short-rate models, capturing many features of the data, but it requires a brute-force approach to produce yield curves, such as simulating a corresponding risk-neutral process out 30 years for each scenario.

For this study Model A is in the extended affine family of multi-factor short-rate models that takes the A23 model of Duffie and Singleton (1999) combined with the extended affine models of Cheridito et al (2007) for the market price of risk. Feldhütter (2008) has a good summary of the details of the extended-affine class, as well as a further expansion to the semi-affine models of Duarte (2004), which Feldhütter (2008) shows give better fits to higher moments of the yields.

3.2 Model B

Model B is the two-factor Black-Karasinski (BK2) model derived from Black and Karisinski (1991). In it, the log of the short-rate is a diffusion that reverts to a temporary mean, which in turn reverts to a long-term mean. It can be expressed as:

$$\begin{aligned} d\,ln\,r(t) &= c\,[b(t) - ln\,r(t)]\,dt + vdB_r(t) \\ db(t) &= j\,[q - b(t)]\,dt + wdB_b(t). \end{aligned}$$

We test alternative calibrations of Model B. Calibration B1 calibrates interest-rate volatility to options prices so it is more or less a risk-neutral calibration. This may be problematic in our real-world context. B3 is a calibration to recent observed levels of volatility for Treasury, and alternative calibration of yield spreads.

For both these calibrations, the yield spreads are modeled by a transition matrix method, independently of interest rates. The vendor has a standard calibration which is used here but an alternative calibration is being developed internally.

3.3 Model L

This is the Libor Market Model (LMM), which models the dynamics of the entire forward curve. The vendor has provided a calibration for US only but we test it for US and UK. The same calibration for yield spreads is used as in model B, and this calibration will be labeled L1. L2 will refer to the alternative calibration of yield spreads.

4 Test results – US data

4.1 Conditional testing

The historical statistics that serve as a basis for the tests are long-term values, but the tests are based on simulated scenarios for the upcoming year. In this case the starting point is the yield curves at 1/1/12 and the projected scenarios are for one year from that. The current short-rates are low but the yield curves are fairly steep. The upcoming year is anticipated to see a continuation of the low-rate environment, but that is not certain. There is little room for decline in interest rates but a lot of upside movement possible. Thus the scenarios would be anticipated to show more skewness than historical data.

Average yields and spreads would be anticipated to be below long-term averages even a year from now, and less than historical volatility seems likely as well. On the other hand, relationships among yields – slope of the curve, yield spreads, credit spreads, etc. – have no apparent reason to vary from historical norms. The conditional spreads and twist given the short rate also have identifiable patterns – slope and especially volatility – that have been fairly consistent over time.

The data used for fitting the historical short-rate model were taken from Federal Reserve Board (Treasury) and Bloomberg (Industrial Bond). The current yield curves for calibrating current factor values and the market prices of risk were also taken from Federal Reserve Board and Bloomberg.

4.2 Moment-based tests

The actual credit spreads might have different mean levels in the projection period than historically due to prevailing economic conditions. In fact both models are projecting spreads a bit higher than historical averages. However we would expect that longer spreads would be higher. This is the case for the projections from Model A but not for Model B. Also the Model A projections are closer to history. See Table 4 for the details.

Standard deviations of yields generally are smaller for the longer maturities. Model A projects this pattern but Model B does not, which can be seen in Table 5. Both models tend to show lower standard deviations than historical. That would make sense given that the movements during a year would be expected to be less than those over a longer historical period.

A large part of the change in corporate yields is due to the underlying change in Treasury yields. Thus the risky and risk-free yields should show a high correlation, less for BBB bonds than for AA. This is indeed seen in the data in Table 6. Since we are moving from a period of very low rates, the big movements can only be upwards, so in this period even higher correlations than historically observed would be expected. This occurs in the

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Table 4: US Average spreads for industrial bonds by maturity

Maturity	0.25	1	3	10	30
AA Empirical	0.55%	0.58%	0.69%	0.79%	1.18%
AA Model A	0.56%	0.68%	0.91%	1.25%	1.54%
AA Model B1	0.91%	1.34%	1.98%	2.00%	1.35%
AA Model B3 L2	0.29%	0.39%	0.65%	1.29%	1.59%
BBB Empirical	1.23%	1.25%	1.43%	1.58%	2.09%
BBB Model A	1.24%	1.40%	1.69%	2.14%	2.52%
BBB Model B1	3.32%	4.10%	3.98%	2.73%	1.59%
BBB Model B3 L2	1.29%	1.46%	1.89%	2.00%	1.82%

Table 5: US Standard deviation by maturity

Maturity	0.25	1	3	10	30
Treasury Empirical	1.44%	1.35%	1.08%	0.69%	0.59%
Treasury Model A	0.64%	0.59%	0.55%	0.54%	0.26%
Treasury Model B1	0.36%	0.40%	0.62%	1.49%	1.78%
Treasury Model B3	1.61%	2.20%	1.50%	1.18%	0.65%
Treasury Model L2	0.50%	0.50%	0.53%	0.64%	0.58%
AA Empirical	1.39%	1.34%	1.11%	0.78%	0.62%
AA Model A	0.66%	0.57%	0.55%	0.57%	0.26%
AA Model B1	0.61%	0.94%	1.30%	1.72%	1.83%
AA Model B3	1.61%	2.20%	1.53%	1.26%	0.68%
AA Model L2	0.52%	0.53%	0.62%	0.78%	0.62%
BBB Empirical	1.33%	1.33%	1.12%	0.90%	0.72%
BBB Model A	0.66%	0.57%	0.55%	0.58%	0.26%
BBB Model B1	1.83%	2.20%	1.82%	1.76%	1.83%
BBB Model B3	1.69%	2.27%	1.62%	1.28%	0.67%
BBB Model L2	0.71%	0.77%	0.83%	0.82%	0.61%

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Model A projections. Whether or not this is too strong for BBB bonds is an open question. Model B does not appear consistent with historical or reasonably anticipated movements.

Table 6: US Treasury correlations with industrial bond yields by maturity

Maturity	0.25	1	3	10	30
AA Empirical	97%	99%	98%	96%	88%
AA Model A	98%	99%	99%	100%	100%
AA Model B1	59%	44%	49%	89%	0.99%
AA Model B3	100%	100%	98%	94%	96%
AA Model L2	97%	94%	86%	82%	95%
BBB Empirical	92%	94%	91%	83%	75%
BBB Model A	97%	98%	98%	99%	99%
BBB Model B1	20%	19%	36%	87%	99%
BBB Model B3	95%	97%	92%	92%	96%
BBB Model L2	69%	64%	63%	78%	95%

4.3 Curve-shape tests

The slope of the conditional 10-year–3-year spread given the 3-month rate and the mean squared errors from this trend line for all the observations in the generated scenarios are compared to empirical. A close match would seem desirable, as the relationships were observed to hold for various historical sub-periods. Higher moments are also compared but here it is unclear what to expect, as the beginning base is very low, which could increase higher moments in the projection period.

Table 7 shows that Model A is reasonably close to the history for all three bonds in both slope and MSE while Model B is not particularly close in either. Neither model matches the empirical higher moments, but Model A appears to be a bit closer. The same comparisons for the conditional 30-year–10-year spread are in Table 8. The slopes for the risky bonds given by Model A are not as close to history as they were for the 10–3 spreads, but the rest of the observations there hold here as well. The higher moments here are more of a toss-up. The comparison for the twist, in Table 9, look quite similar to those for the 10–3 spread.

Figure 3 and 4 show the standard deviation of curvature, and standard deviation of annual change of curvature of different maturities.

5 Test results – UK data

5.1 Background

Models A and B were also fit to UK data. Model B was first developed for UK data and performs better for it than for the US data. The curve-shape tests here look good for the risk-free rates for both models. Model B still has some problems with moments and risky bonds however.

5.2 Moment-based tests

As in the US, the projected credit spreads for a year ahead in Table 10 are higher than historical, due to the current economic conditions. Nonetheless Model B results look unusual, in part due to the decreasing spreads at longer maturities.

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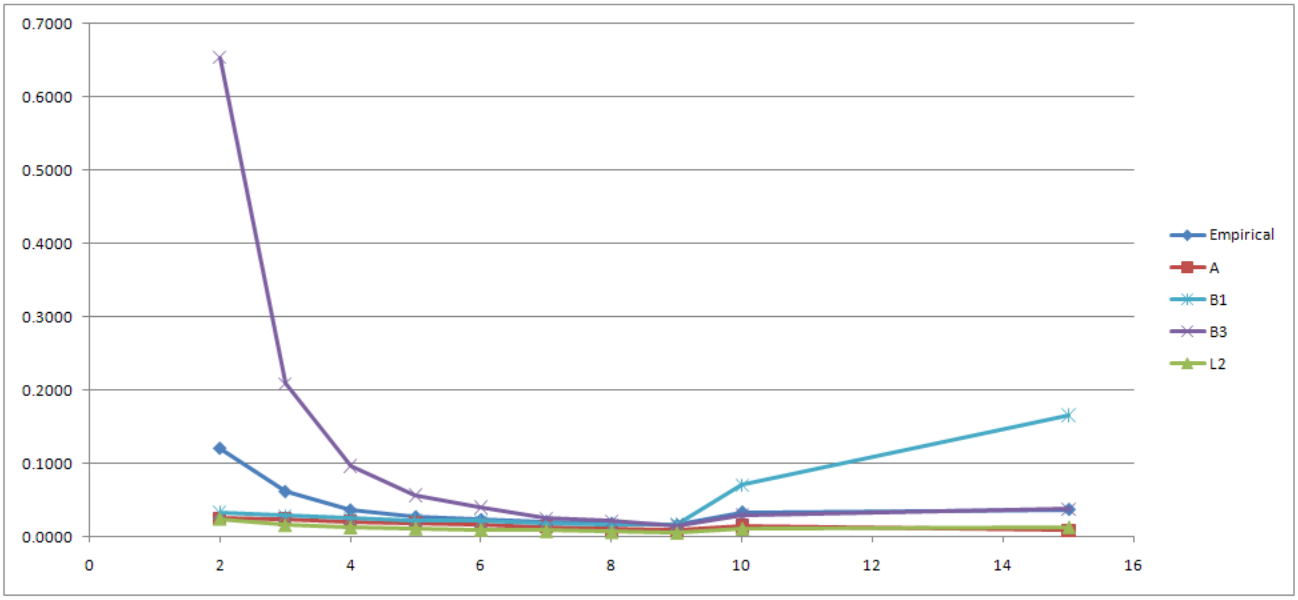

Figure 3: US standard deviation of curvature by maturities

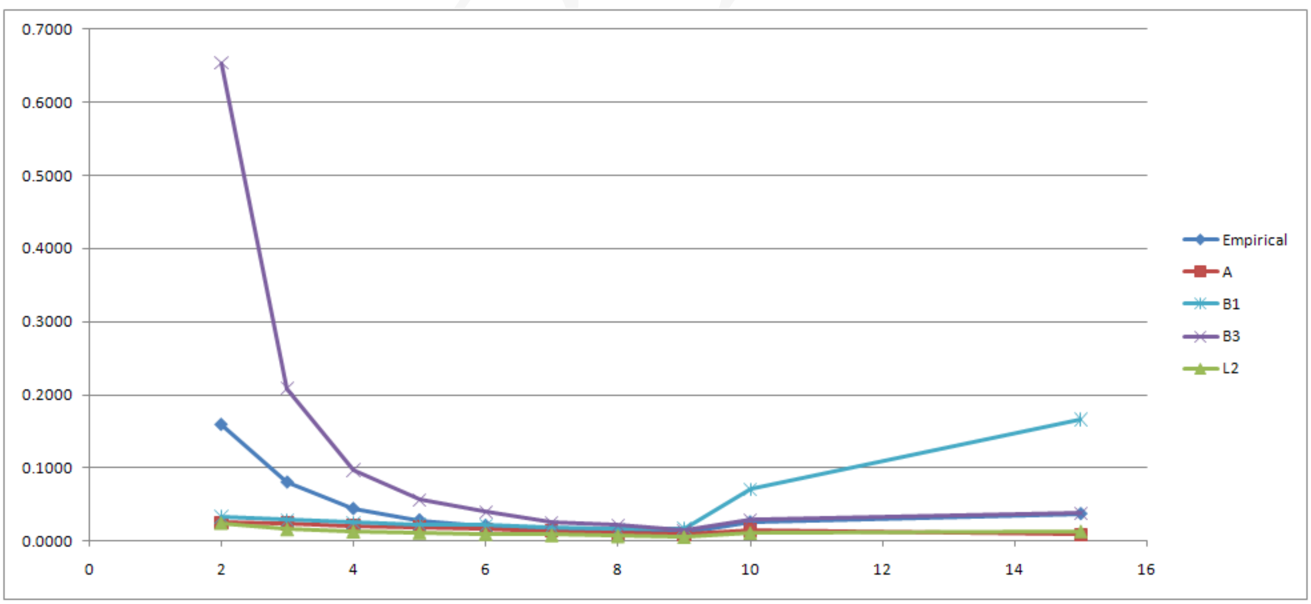

Figure 4: US standard deviation of annual change of curvature by maturities

Table 7: US Comparison of conditional distribution of 10-year to 3-year spread given 3-month rate

	Slope	MSE	Skewness	Kurtosis	Trend Line
Treasury Empirical	-0.3909	0.0029	-0.3880	0.1558	y=-0.3909x+0.0236
Treasury Model A	-0.3865	0.0020	-1.9530	7.3433	y=-0.3865x+0.0190
Treasury Model B1	0.9406	0.0101	3.0586	25.6156	y= 0.9406x+0.0113
Treasury Model B3	-0.4179	0.0052	0.5773	4.5184	y= -0.4179x+0.0164
Treasury Model L2	-0.3452	0.0029	0.5420	1.1010	y= -0.3452x+0.0146
AA Empirical	-0.4159	0.0031	-0.0791	0.0615	y=-0.4159x+0.0277
AA Model A	-0.4092	0.0015	-1.4230	4.7026	y=-0.4092x+0.0251
AA Model B1	-0.1087	0.0113	3.2490	25.6853	y=-0.1087x+0.0174
AA Model B3	-0.4100	0.0055	0.3885	2.0269	y=-0.4100x+0.0240
AA Model L2	-0.2733	0.0034	0.3291	0.4927	y=-0.2733x+0.0213
BBB Empirical	-0.3892	0.0032	0.6397	2.4337	y=-0.3892x+0.0298
BBB Model A	-0.4106	0.0013	-1.2535	3.8440	y=-0.4106x+0.0290
BBB Model B1	-0.4029	0.0115	3.5415	27.7891	y=-0.4029x+0.0185
BBB Model B3	-0.4027	0.0053	0.3744	1.4330	y=-0.4027x+0.0227
BBB Model L2	-0.2941	0.0030	0.5420	1.0634	y=-0.2941x+0.0192

Table 8: US Comparison of conditional distribution of 30-year to 10-year spread given 3-month rate

	Slope	MSE	Skewness	Kurtosis	Trend Line
Treasury Empirical	-0.1288	0.0035	-0.9306	2.7547	y=-0.1288x+0.0078
Treasury Model A	-0.1492	0.0027	1.5382	4.7477	y=-0.1492x+0.0079
Treasury Model B1	-1.0787	0.0077	0.0979	6.7758	y=-1.0787x+0.0124
Treasury Model B3	-0.2145	0.0055	1.4179	90.4816	y=-0.2145x+0.0100
Treasury Model L2	-0.3366	0.0003	0.8932	8.2794	y=-0.3366x+0.0069
AA Empirical	-0.1533	0.0029	0.2351	0.5335	y=-0.1533x+0.0134
AA Model A	-0.0609	0.0031	1.2970	3.9583	y=-0.0609x+0.0097
AA Model B1	-1.0495	0.0078	0.0720	7.1758	y=-1.0495x+0.0153
AA Model B3	-0.2237	0.0059	1.7053	92.1069	y=-0.2237x+0.0137
AA Model L2	-0.4222	0.0020	-0.3860	-0.0533	y=-0.0422x+0.0117
BBB Empirical	-0.1054	0.0035	0.0840	0.9082	y=-0.1054x+0.0135
BBB Model A	-0.0321	0.0032	1.2026	3.6611	y=-0.0321x+0.0102
BBB Model B1	-0.3535	0.0083	-0.1526	9.9871	y=-0.3535x+0.0093
BBB Model B3	-0.2522	0.0059	4.0590	210.82	y=-0.2522x+0.0116
BBB Model L2	-0.4875	0.0011	0.6226	1.1324	y=-0.4875x+0.0124

Table 9: US Comparison of conditional distribution of twist given 3-month rate

	Slope	MSE	Skewness	Kurtosis	Trend Line
Treasury Empirical	-0.0494	0.0004	0.1320	2.8125	y=-0.0494x+0.0030
Treasury Model A	-0.0478	0.0004	-1.7919	6.1934	y=-0.0478x+0.0023
Treasury Model B1	0.1883	0.0013	5.7359	76.7443	y= 0.1883x+0.0010
Treasury Model B3	-0.0490	0.0008	0.4299	0.2394	y= -0.0490x+0.0018
Treasury Model L2	-0.0325	0.0004	0.5546	1.1293	y= -0.0325x+0.0017
AA Empirical	-0.0518	0.0004	0.2625	1.7039	y=-0.0518x+0.0033
AA Model A	-0.0554	0.0004	-1.3645	4.3281	y=-0.0554x+0.0031
AA Model B1	0.0370	0.0015	5.5085	65.5806	y= 0.0370x+0.0017
AA Model B3	-0.0474	0.0009	0.3344	0.1458	y= -0.0474x+0.0027
AA Model L2	-0.0179	0.0005	0.2351	0.2285	y= -0.0179x+0.0025
BBB Empirical	-0.0503	0.0005	1.2049	5.0620	y=-0.0503x+0.0036
BBB Model A	-0.0570	0.0004	-1.2318	3.7639	y=-0.0570x+0.0036
BBB Model B1	0.0399	0.0016	5.8725	64.2966	y= 0.0399x+0.0022
BBB Model B3	-0.0449	0.0009	0.3772	0.2415	y= -0.0449x+0.0027
BBB Model L2	-0.0176	0.0004	0.4935	0.9795	y= -0.0176x+0.0021

The standard deviations of the projected yields are in Table 11. Model A follows empirical in showing declining volatility of yields as maturity increases. It has less volatility than empirical but as this is conditional on starting values, that seems appropriate. Nonetheless the longest volatilities look surprisingly low. Model B for sovereigns shows flat volatilities, which is an improvement over the increasing volatilities it has for the US. The risky volatilities do decline for longer bonds, but are unusually high.

Table 10: UK Average spreads for industrial bonds by maturity

Maturity	0.25	1	3	10	20
AA Empirical	0.46%	0.48%	0.54%	0.82%	0.91%
AA Model A	0.56%	0.82%	1.26%	1.63%	1.50%
AA Model B1	2.46%	4.16%	4.89%	3.05%	1.88%
AA Model B3 L2	0.29%	0.55%	1.01%	1.52%	1.53%
BBB Empirical	0.96%	1.02%	1.17%	1.61%	1.69%
BBB Model A	1.10%	1.50%	2.19%	2.69%	2.35%
BBB Model B1	8.53%	9.29%	7.14%	3.67%	2.19%
BBB Model B3 L2	1.39%	1.71%	2.14%	2.16%	1.87%

The correlations between risky and risk-free yields closely match empirical for Model A but not for Model B.

5.3 Curve-shape tests

Figure 5 and 6 show the standard deviation of curvature, and standard deviation of annual change of curvature of different maturities.

For the $10-3$ spreads in Table 13, Model A closely matches the historical slopes and MSEs. Model B is showing slopes closer to empirical than it has in the US but still a bit removed. The volatilities continue to be too high. Model B is more consistent with history than is Model A on the higher moments. Table 14 has the comparable $20-10$ year spreads. These are about equally far away from empirical for the two models, but in opposite directions.

Table 11: UK Standard deviation by maturity

Maturity	0.25	1	3	10	20
Sovereign Empirical	1.30%	1.23%	0.96%	0.68%	0.64%
Sovereign Model A	0.89%	0.94%	0.99%	0.48%	0.21%
Sovereign Model B1	1.07%	1.05%	0.94%	1.01%	1.00%
Sovereign Model B3	0.79%	0.80%	0.82%	0.63%	0.67%
Sovereign Model L2	0.66%	0.64%	0.62%	0.69%	0.70%
AA Empirical	1.31%	1.22%	0.94%	0.56%	0.50%
AA Model A	0.95%	0.96%	0.92%	0.31%	0.11%
AA Model B1	2.32%	3.70%	3.55%	1.93%	1.30%
AA Model B3	0.80%	0.85%	0.94%	0.78%	0.73%
AA Model L2	0.67%	0.69%	0.76%	0.82%	0.75%
BBB Empirical	1.16%	1.07%	0.89%	0.63%	0.69%
BBB Model A	0.99%	0.98%	0.87%	0.24%	0.08%
BBB Model B1	6.66%	6.17%	4.13%	1.98%	2.31%
BBB Model B3	0.94%	1.04%	1.13%	0.81%	0.73%
BBB Model L2	0.82%	0.91%	0.98%	0.85%	0.75%

Table 12: UK Sovereign correlations with industrial bond yields by maturity

Maturity	0.25	1	3	10	20
AA Empirical	99%	99%	99%	97%	98%
AA Model A	99%	99%	99%	98%	94%
AA Model B1	47%	29%	27%	53%	78%
AA Model B3	99%	95%	88%	81%	92%
AA Model L2	98%	93%	81%	83%	93%
BBB Empirical	95%	96%	95%	92%	95%
BBB Model A	97%	98%	99%	92%	83%
BBB Model B1	17%	18%	24%	52%	78%
BBB Model B3	85%	78%	74%	78%	93%
BBB Model L2	80%	70%	63%	81%	93%

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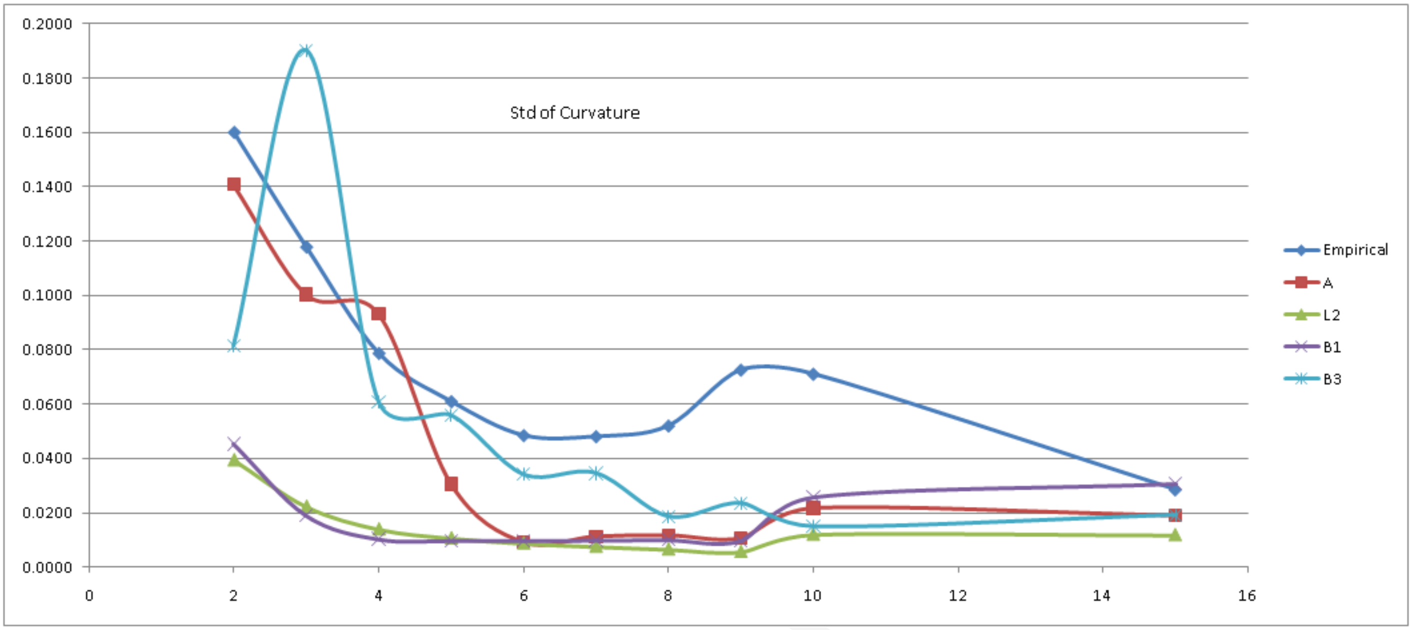

Figure 5: UK standard deviation of curvature by maturities

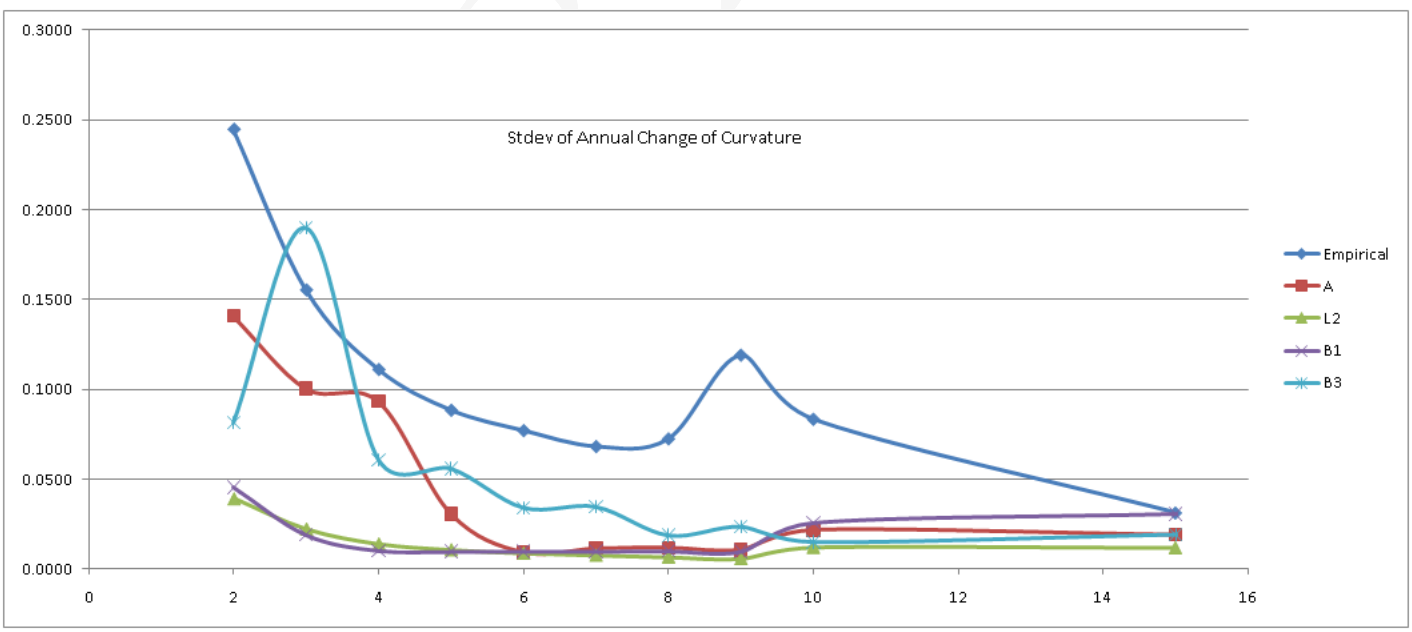

Figure 6: UK standard deviation of annual change of curvature by maturities

Table 13: UK Comparison of conditional distribution of 10-year to 3-year spread given 3-month rate

	Slope	MSE	Skewness	Kurtosis	Trend Line
Sovereign Empirical	-0.4792	0.0025	-0.5840	0.9666	y=-0.4792x+0.0220
Sovereign Model A	-0.5018	0.0025	-1.5009	6.3080	y=-0.5018x+0.0248
Sovereign Model B1	-0.2133	0.0055	0.9448	1.6727	y= -0.2133x+0.0134
Sovereign Model B3	-0.5216	0.0038	1.4583	17.8978	y= -0.5216x+0.0152
Sovereign Model L2	-0.3279	0.0029	0.6442	1.3507	y= -0.3279x+0.0151
AA Empirical	-0.5322	0.0029	0.6512	-0.1132	y=-0.5322x+0.0290
AA Model A	-0.5707	0.0029	-0.6691	2.6963	y=-0.5707x+0.0330
AA Model B1	-0.7332	0.0086	0.8289	1.6106	y=-0.7332x+0.0207
AA Model B3	-0.4985	0.0039	1.0682	9.6117	y=-0.4985x+0.0217
AA Model L2	-0.3146	0.0030	0.5839	1.2105	y=-0.3146x+0.0209
BBB Empirical	-0.5158	0.0032	1.0380	1.9130	y=-0.5158x+0.0329
BBB Model A	-0.5922	0.0029	-0.5323	2.3896	y=-0.5922x+0.0381
BBB Model B1	-0.3456	0.0068	0.8864	1.4206	y=-0.3456x+0.0102
BBB Model B3	-0.5165	0.0038	1.3710	15.872	y=-0.5165x+0.0225
BBB Model L2	-0.3969	0.0030	0.5458	1.1557	y=-0.3969x+0.0218

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Table 14: UK Comparison of conditional distribution of 20-year to 10-year spread given 3-month rate

	Slope	MSE	Skewness	Kurtosis	Trend Line
Sovereign Empirical	-0.2320	0.0016	-0.3302	0.3450	y=-0.2320x+0.0094
Sovereign Model A	-0.2335	0.0018	-1.6509	6.8242	y=-0.2335x+0.0126
Sovereign Model B1	-0.2672	0.0022	0.6820	1.2414	y=-0.2672x+0.0100
Sovereign Model B3	-0.1809	0.0018	1.9082	33.5899	y=-0.1809x+0.0092
Sovereign Model L2	-0.1994	0.0006	0.5789	1.8719	y=-0.1994x+0.0090
AA Empirical	-0.2158	0.0021	0.2962	-0.4151	y=-0.2158x+0.0101
AA Model A	-0.1881	0.0011	-1.6133	7.4610	y=-0.1881x+0.0115
AA Model B1	-0.3679	0.0032	0.9686	4.4852	y=-0.3679x+0.0088
AA Model B3	-0.2123	0.0024	1.7171	37.270	y=-0.2123x+0.0101
AA Model L2	-0.2401	0.0016	-0.213	-0.0307	y=-0.2401x+0.0105
BBB Empirical	-0.1174	0.0024	0.0475	0.0078	y=-0.1174x+0.0061
BBB Model A	-0.1427	0.0007	-1.8056	9.7736	y=-0.1427x+0.0089
BBB Model B1	-0.1360	0.0034	1.1004	3.5357	y=-0.1360x+0.0049
BBB Model B3	-0.2620	0.0022	5.7913	188.62	y=-0.2620x+0.0102
BBB Model L2	-0.2946	0.0012	0.2147	0.2234	y=-0.2946x+0.0117

For the shape twist in Table 15, Model A is closer to empirical but Model B is reasonable.

Table 15: UK Comparison of conditional distribution of twist between 20-to-10 year slope and 10-to-3 year slope given 3-month rate

	Slope	MSE	Skewness	Kurtosis	Trend Line
Sovereign Empirical	-0.0453	0.0003	-0.8393	3.8997	y=-0.0453x+0.0022
Sovereign Model A	-0.0483	0.0002	-1.0197	4.6067	y=-0.0483x+0.0023
Sovereign Model B1	-0.0037	0.0006	1.0586	3.3013	y= -0.0037x+0.0009
Sovereign Model B3	-0.0551	0.0004	1.1661	7.9500	y= -0.0551x+0.0012
Sovereign Model L2	-0.0269	0.0004	0.6478	1.4773	y= -0.0269x+0.0012
AA Empirical	-0.0544	0.0004	-0.2371	0.0352	y=-0.0544x+0.0031
AA Model A	-0.0627	0.0003	-0.4182	1.6270	y=-0.0627x+0.0036
AA Model B1	-0.0680	0.0010	0.8586	1.8783	y= -0.0680x+0.0021
AA Model B3	-0.0500	0.0005	0.3718	0.9886	y= -0.0816x+0.0019
AA Model L2	-0.0209	0.0004	0.3092	0.7831	y= -0.0500x+0.0021
BBB Empirical	-0.0619	0.0005	0.3468	-0.2386	y=-0.0619x+0.0041
BBB Model A	-0.0731	0.0004	-0.3221	1.4339	y=-0.0731x+0.0055
BBB Model B1	-0.0358	0.0008	1.0642	1.9628	y= -0.0358x+0.0010
BBB Model B3	-0.0476	0.0004	0.5930	1.1205	y= -0.0476x+0.0022
BBB Model L2	-0.0272	0.0004	0.6377	1.3661	y= -0.0272x+0.0019

6 Concluding comments

A number of tests for simulated yield-curve scenarios were assembled from the literature and some of them were expanded a bit in scope. They were applied to two models of US and UK yield curves – Model B for which output and documentation came from the vendor and some internal corporate sources, and Model A which was put together from articles in recent literature as a basis of comparison. Model A matches well to the historical stylized facts for the US and reasonably well for the UK. Model B does not match well at all to US data but is somewhat better in the UK. Its credit-spread component appears to be more problematic than its risk-free component and indeed the vendor is working to improve that area. This model could actually work quite well for options pricing, which seems to be where it arose, but that was not tested.

Model A incorporates some features in the contemporary literature, such as: modeling correlation of factors, including stochastic volatility, as pioneered by Dai and Singleton (2000); the simultaneous modeling of risky and risk-free bonds as in Duffie and Singleton (1999); and the extended affine models of Cheridito et al. (2007). There are other potentially useful recent developments not incorporated, such as the stochastic-volatility affine jump-diffusion model of Andersen et al. (2004) and the semi-affine model of Duarte (2004). The tests indicate that there is room for improvement in both models, and the literature provides some avenues of improvement.

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